A Minimal Conserved Recursive-Survival Sector for Early Structural Maturation

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Canonical v0.2 – expanded working draft

Abstract

This note introduces a minimal conserved Recursive-Survival sector for modelling early structural maturation in FLRW cosmology. The aim is not to replace general relativity, but to add a disciplined effective fluid representing unresolved survival-weighted histories that have not yet settled into the ordinary classical description. The total stress-energy remains covariantly conserved. Ordinary represented density and recursive-sector density exchange through a current aligned with the cosmological four-velocity. In the homogeneous reduction, detailed survival exposure is absorbed into an effective exchange coefficient and a coherence-memory variable. The resulting system evolves ordinary density, recursive density, expansion, and coherence per e-fold. The model is designed to recover standard GR when recursive density and coherence decay. Its first purpose is numerical: to test whether a transient recursive sector can enhance early represented structural maturation without leaving a large late-time remnant.

1 Plain-Language Version Before the Mathematics

This section gives the model in ordinary language before introducing the formal equations. The purpose is to let the reader know what the mathematics is trying to express.

1.1 The Basic Idea

Standard cosmology describes how the universe expands and how ordinary components such as radiation, matter, and vacuum energy affect that expansion. Recursive Survival adds one extra interpretive ingredient: not every possible history is represented equally after loss. Some histories preserve coherence, closure, and structure better than others. Those better-preserved histories can become more strongly represented without requiring the universe to compute or store every alternative in full detail.

The model below turns that idea into a deliberately small effective sector. The ordinary cosmological sector is the part already described by standard cosmology. The recursive-survival sector is a temporary bookkeeping sector for unresolved survival-weighted structure. It is not meant to be a new magic substance. It is a way of asking whether unresolved recursive structure could briefly influence early represented structure and then fade away.

1.2 Why Conservation Matters

The most important rule is that the total ledger must close. If the ordinary sector gains effective density from the recursive sector, the recursive sector loses the corresponding amount. The model therefore does not simply add an arbitrary density to the universe. It uses two sectors whose total stress-energy remains conserved.

This is the reason for the word *conserved* in the title. It signals that the recursive term is constrained by the same conservation discipline that ordinary relativistic cosmology uses.

1.3 The Three Dynamical Ingredients

The minimal model tracks three main quantities.

First, it tracks the recursive density fraction. This tells us how much of the total effective density is still in the recursive-survival sector. If this fraction stays large at late times, the model fails.

Second, it tracks coherence memory. This measures whether the recursive sector is still coherent enough to influence the ordinary represented sector. The coherence memory is bounded between zero and one, so it behaves like a genuine state variable rather than an unlimited source.

Third, it tracks exchange. Exchange is the rate at which coherent recursive sector density is transferred into the ordinary represented sector. The exchange is present only when recursive density and coherence are both present.

1.4 The Intended Story

In the early universe, the recursive sector may be present and coherent enough to matter. During that window, it can transfer effective density into the ordinary represented sector and produce a transient boost in represented structural maturation.

As the universe evolves, recursive density is depleted and coherence memory decays. The exchange shuts off. The late universe then approaches ordinary FLRW/GR behaviour. The model is interesting only if this early window appears without leaving a large late-time remnant.

1.5 What the Mathematics Will Do

The equations below do four jobs:

(i) enforce total conservation between the ordinary and recursive sectors;

(ii) define a minimal exchange law between the sectors;

(iii) define a bounded coherence-memory law;

(iv) reduce the model to a small system that can be integrated numerically.

The reader should not interpret the equations as claiming that the universe is literally solving these lines of mathematics. The equations are an effective smooth description of the relationships among density, coherence, exchange, and expansion after projecting the richer Recursive-Survival picture onto a homogeneous cosmological layer.

2 Purpose of the Minimal Model

The early universe may appear structurally mature without requiring infinite processing or exhaustive selection over all possibilities. In ordinary cosmology, structure grows through gravity, cooling, collapse, feedback, mergers, and selection effects. Recursive Survival adds a complementary interpretation: generated histories are not represented equally after loss. Histories that preserve structure, coherence, closure, or low-loss propagation better than others may dominate the represented record earlier than a naive equal-weight picture would suggest.

The central modelling question is whether this idea can be made into a small, controlled cosmological sector rather than an unconstrained addition. The proposed answer is to treat unresolved recursive survival structure as an effective sector with its own stress-energy, exchange law, coherence memory, and decay mechanism.

The word *conserved* in the title is deliberate. It says that the recursive-survival sector is not a free density bolted onto FLRW. It is a two-sector effective description in which the total stress-energy ledger closes. The ordinary and recursive sectors may exchange density, but the sum obeys the Bianchi identity.

2.1 What the Model Claims

The model claims the following, and no more:

(i) A transient recursive-survival sector can be represented, at lowest homogeneous order, as an effective fluid.

(ii) The ordinary represented sector and recursive sector may exchange effective density through a covariant current.

(iii) The exchange can be made proportional to recursive abundance, coherence memory, expansion activity, and an effective survival-exposure coefficient.

(iv) The sector can be arranged to decay, leaving ordinary FLRW evolution at late times.

(v) The idea becomes meaningful only if a compact parameter range gives a transient early effect without a large late-time remnant.

2.2 What the Model Does Not Claim

This note does not claim a completed microscopic derivation, a full perturbation theory, or a replacement for general relativity. It does not claim that the universe computes every possible history. It also does not assert that the ordinary differential equations below act directly on bare topological supports. Rather, the equations are a projected, smooth, lowest-mode description suitable for first numerical tests.

In Austin's language, the equations express relationships in the projected effective layer. They do not control the physical process itself.

3 Notation and Conventions

Throughout this note set $c = 1$. This is a standard simplification meaning that the speed of light is treated as one unit, so factors of c do not clutter the equations. Greek indices such as μ and ν label spacetime directions. Latin indices such as i and j label spatial directions. The ordinary represented sector has density ρ and pressure $p = w\rho$. The recursive sector has density ρ_R and pressure $p_R = w_R\rho_R$. The scale factor is $a(t)$, the Hubble rate is $H = \dot{a}/a$, and the e-fold time is

$$N = \ln a. \tag{1}$$

Primes denote differentiation with respect to N:

$$X' = \frac{\mathrm{d}X}{\mathrm{d}N} = \frac{1}{H}\frac{\mathrm{d}X}{\mathrm{d}t}. \tag{2}$$

3.1 How to Read the Main Notation

A dot above a symbol means change with respect to ordinary time t. For example, $\dot{a}$ means the rate of change of the scale factor. A prime means change with respect to e-fold time N. For example, C' means the rate of change of coherence per e-fold. A subscript R marks a recursive-sector quantity. The symbol ∇_μ means a covariant derivative: the relativistic version of taking a derivative in curved spacetime.

Table 1: Novice-facing notation guide for the minimal Recursive-Survival sector.

Symbol	Short meaning	Plain-language explanation
c	Speed of light	Set to 1 to simplify units. This does not remove relativity; it only avoids writing repeated factors of c.
π	Circle constant	The usual mathematical constant appearing in Einstein's equation through $8\pi G$.
G	Newton's gravitational constant	Sets the strength of gravity in Einstein's equation. It is not the same symbol as $G_{\mu\nu}$.
$g_{\mu\nu}$	Spacetime metric	The object that describes distances, times, and curvature.
$g^{\mu\nu}$	Inverse metric	The inverse of $g_{\mu\nu}$, used to raise tensor indices.
$G_{\mu\nu}$	Einstein curvature tensor	The left-hand side of Einstein's equation. It encodes spacetime curvature.
μ, ν	Spacetime indices	Labels for time and space directions in tensor equations. Repeated indices are summed in the usual tensor convention.
i, j	Spatial indices	Labels for spatial directions only.
δ_{ij}	Flat spatial metric symbol	Equals 1 when $i = j$ and 0 otherwise; used for a flat spatial FLRW metric.
∇_μ	Covariant derivative	The derivative appropriate for curved spacetime. It respects the metric geometry.
$T_{\mu\nu}$	Ordinary stress-energy	The energy, pressure, and flow of the ordinary represented sector.
$T^{\mu\nu}$	Raised-index ordinary stress-energy	The same ordinary stress-energy tensor with indices raised using the metric.
$T^R_{\mu\nu}$, $T_R^{\mu\nu}$	Recursive stress-energy	The effective stress-energy assigned to the recursive-survival sector. The superscript/subscript R labels the recursive sector.
Q^ν	Exchange current	A covariant bookkeeping current that says how stress-energy moves between the ordinary and recursive sectors.

Symbol	Short meaning	Plain-language explanation
u^ν, u^μ	Four-velocity	The relativistic velocity of the cosmological fluid. In FLRW, it points along the common cosmic rest frame.
Ξ	Exchange scalar	The homogeneous exchange rate. If $\Xi > 0$, recursive density is transferred into the ordinary sector.
ρ	Ordinary density	The effective density of the ordinary represented sector in the simplified model.
p	Ordinary pressure	Pressure of the ordinary represented sector.
w	Ordinary equation-of-state parameter	Defined by $p = w\rho$. It tells how pressure compares with density.
ρ_R	Recursive density	The effective density stored in unresolved recursive-survival structure.
p_R	Recursive pressure	Pressure assigned to the recursive effective fluid.
w_R	Recursive equation-of-state parameter	Defined by $p_R = w_R\rho_R$. It controls how the recursive sector redshifts with expansion.
ρ_{tot}	Total density	The sum $\rho + \rho_R$. It is the total density sourcing the background expansion in the minimal model.
ρ_r	Radiation density	The usual radiation component in a more realistic split of the ordinary sector.
ρ_b	Baryon density	Ordinary matter made of baryons in a realistic cosmology.
ρ_c	Cold dark matter density	The cold dark matter component in a realistic cosmology.
ρ_Λ	Vacuum density	The cosmological-constant or vacuum-energy component.
$a(t)$	Scale factor	Measures the relative size of the universe as a function of cosmic time.
t	Cosmic time	The ordinary time variable used in FLRW cosmology.
τ	Proper time / path time	The time parameter used inside the survival-exposure integral.
H	Hubble rate	$H = \dot{a}/a$. It measures the expansion rate of the universe.
H_{standard}	Standard comparison Hubble rate	The expansion rate in the baseline model without the recursive sector.
N	E-fold time	$N = \ln a$. It is a convenient time variable for cosmology; one e-fold means the scale factor has multiplied by e.

Symbol	Short meaning	Plain-language explanation
N_i, N_f	Initial and final e-folds	The beginning and end of a numerical run.
$\dot{X}$	Time derivative	Means $\mathrm{d}X/\mathrm{d}t$, the rate of change with respect to cosmic time.
X'	E-fold derivative	Means $\mathrm{d}X/\mathrm{d}N$, the rate of change per e-fold of expansion.
d	Differential symbol	The small-change symbol used in integrals and derivatives, such as $\mathrm{d}A$ or $\mathrm{d}t$.
ln	Natural logarithm	The logarithm with base e. Used in $N = \ln a$ and $A = -\ln S$.
S	Survival weight	A weight representing how strongly a history survives or remains represented.
A	Accumulated survival loss	Defined by $A = -\ln S$. Larger A means more accumulated loss.
ΓW	Survival-loss exposure	A compact symbol for the effective rate of survival filtering or loss exposure.
s_μ	Survival-exposure gradient	$s_\mu = \nabla_\mu A$. It points in the spacetime direction where accumulated survival loss changes.
$\mathcal{E}$	Exposure per e-fold	$\mathcal{E} = \mathrm{d}A/\mathrm{d}N = \Gamma W/H$. A dimensionless way to track survival exposure in e-fold time.
$\mathcal{E}_{\text{eff}}$	Effective exposure value	A simplified or averaged exposure absorbed into the minimal coupling.
α_0	Bare exposure coupling	Coupling used when exposure $\mathcal{E}$ is written explicitly.
α_R	Minimal exchange strength	Dimensionless coefficient controlling how strongly coherent recursive density transfers into the ordinary sector.
C	Coherence memory	A bounded variable between 0 and 1 measuring how much coherent recursive structure remains active.
C_0, C_i	Initial coherence	Starting value of the coherence memory.
C_*	Instantaneous coherence equilibrium	The value C tends toward if Ω_R is held fixed.
γ_C	Coherence decay per e-fold	Controls how quickly C fades when it is not sourced.
η	Coherence source strength	Controls how strongly recursive abundance sources coherence memory.
Ω_R	Recursive density fraction	$\Omega_R = \rho_R/(\rho + \rho_R)$. It says what fraction of the total density lies in the recursive sector.
$\Omega_{R,i}$	Initial recursive fraction	Starting value of Ω_R for a numerical run.
w_{eff}	Effective equation of state	The combined equation-of-state value of the ordinary and recursive sectors.

Symbol	Short meaning	Plain-language explanation
$\mathcal{M}$	Toy maturation index	A diagnostic variable used to track possible represented structural maturation without claiming a full perturbation theory.
s_R	Maturation response coefficient	Controls how strongly $C\Omega_R$ affects the toy maturation index.
Θ	Parameter set	Shorthand for the model parameters used in a scan, such as $\{\alpha_R, \gamma_C, \eta, w, w_R\}$.
$\tilde{N}$	Dummy integration variable	A placeholder e-fold variable used inside an integral so it is not confused with the upper limit N.
$\hbar$	Reduced Planck constant	The constant that marks explicitly quantum scales. It is not used in the minimal classical effective system, but becomes important in quantum-history extensions.
γ_i	Candidate history	A possible coarse-grained spacetime or recursive history in a quantum-history reading. The index i labels which history is being discussed.
S_i	Survival weight of history i	The survival weight assigned to a particular history γ_i.
A_i	Accumulated loss of history i	The accumulated survival loss associated with a particular history, with $S_i = e^{-A_i}$ in the simplest weighting.
Z	Normalization factor	A partition-function-like quantity used to make weights add up consistently in statistical or quantum-history analogies.
$\hat{T}^{\mu\nu}$	Stress-energy operator	A quantum operator version of stress-energy. A hat marks an operator rather than an ordinary classical quantity.
$\langle\cdot\rangle$	Expectation value	The averaged value of a quantum operator in a chosen state, history ensemble, or coarse-grained sector.
S_{surv}	Survival entropy	Entropy associated with unresolved surviving histories or survival-weighted representation.
S_{grav}	Gravitational entropy	Entropy associated with gravitational degrees of freedom, such as horizons or coarse-grained geometry.
A_{hor}	Horizon area	The area entering horizon-entropy expressions, written separately from survival loss A to avoid confusion.

Symbol	Short meaning	Plain-language explanation
$\delta\rho_R$	Recursive density perturbation	A small fluctuation in the recursive-sector density.
δC	Coherence perturbation	A small fluctuation in the coherence-memory variable.
Φ, Ψ	Metric perturbations	Scalar perturbations of the cosmological metric used in standard perturbation theory.
$[0, 1]$	Unit interval	The allowed range for bounded variables such as C and Ω_R.
$\to$	Tends toward	Used for late-time limits such as $\Omega_R \to 0$.
$\geq, \leq$	Greater/less than or equal to	Used to state allowed parameter ranges and positivity conditions.

4 Two-Sector Conservation

Begin with an effective Einstein equation,

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu} + T^R_{\mu\nu}\right). \tag{3}$$

Here $T_{\mu\nu}$ is the ordinary effective stress-energy and $T^R_{\mu\nu}$ is the recursive-survival sector. Since $\nabla_\mu G^{\mu\nu} = 0$, the Bianchi identity requires total conservation:

$$\nabla_\mu \left(T^{\mu\nu} + T^{\mu\nu}_R\right) = 0. \tag{4}$$

The two sectors may exchange stress-energy through a current Q^ν:

$$\nabla_\mu T^{\mu\nu} = Q^\nu, \tag{5}$$

$$\nabla_\mu T^{\mu\nu}_R = -Q^\nu. \tag{6}$$

Equations (5) and (6) are the basic discipline upgrade. Recursive effects are not free additions; any gain in one sector is balanced by a loss in the other. In the homogeneous background, this exchange will reduce to a scalar Ξ. In a less symmetric setting, Q^ν could include momentum exchange, anisotropic components, or dependence on gradients of survival exposure.

5 Survival Variables

Recursive Survival is more naturally expressed through accumulated loss than through survival weight itself. Define

$$A = -\ln S, \tag{7}$$

where S is survival weight. Schematically,

$$A = \int \Gamma W \, \mathrm{d}\tau, \tag{8}$$

where ΓW is an effective survival-loss exposure. The gradient

$$s_\mu = \nabla_\mu A \tag{9}$$

can be read as the local direction and strength of survival exposure.

In a full tensor model, the recursive-sector stress-energy could depend on

$$T^R_{\mu\nu} = T^R_{\mu\nu}\left[A,\, s_\mu,\, C,\, \text{boundary stability, closure diagnostics},\, g_{\mu\nu}\right]. \tag{10}$$

The present note does not attempt that full construction. It keeps only the lowest homogeneous mode: an effective density ρ_R, an equation of state w_R, and a coherence memory C.

5.1 Exposure Per E-Fold

Using $N = \ln a$, define the dimensionless survival exposure per e-fold

$$\mathcal{E} = \frac{\mathrm{d}A}{\mathrm{d}N} = \frac{\Gamma W}{H}. \tag{11}$$

This makes clear that ΓW has dimensions of inverse time, while $\mathcal{E}$ is dimensionless. It is also the natural quantity for numerical integration in e-fold time.

6 Homogeneous FLRW Reduction

Assume a spatially flat FLRW metric,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j. \tag{12}$$

The ordinary and recursive sectors are treated as perfect fluids at background order:

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p g^{\mu\nu}, \tag{13}$$
$$T^{\mu\nu}_R = (\rho_R + p_R)u^\mu u^\nu + p_R g^{\mu\nu}. \tag{14}$$

Use

$$p = w\rho, \qquad p_R = w_R\rho_R. \tag{15}$$

The first-pass Friedmann equation is

$$H^2 = \frac{8\pi G}{3}\left(\rho + \rho_R\right). \tag{16}$$

Curvature, radiation splitting, baryon/CDM separation, and vacuum density are not excluded. They are suppressed here so the recursive exchange mechanism can be isolated. In a realistic implementation, one should replace the single ordinary sector by

$$\rho \longrightarrow \rho_r + \rho_b + \rho_c + \rho_\Lambda, \tag{17}$$

with each component assigned its usual continuity equation unless a specific recursive coupling is introduced.

7 Exchange Current and Sign Convention

In homogeneous FLRW, choose the exchange current parallel to the cosmological four-velocity:

$$Q^\nu = \Xi u^\nu. \tag{18}$$

The sign convention is

$$\Xi > 0 \iff \text{recursive-sector density is transferred into the ordinary sector.} \tag{19}$$

The continuity equations are then

$$\dot{\rho} + 3H(1+w)\rho = +\Xi, \tag{20}$$
$$\dot{\rho}_R + 3H(1+w_R)\rho_R = -\Xi. \tag{21}$$

Adding (20) and (21) gives the conserved total continuity equation,

$$\dot{\rho}_{\text{tot}} + 3H\left[(1+w)\rho + (1+w_R)\rho_R\right] = 0, \tag{22}$$

where $\rho_{\text{tot}} = \rho + \rho_R$.

8 Minimal Exchange Law

The minimal Austin-style exchange law is

$$\boxed{\Xi = \alpha_R C H \rho_R} \tag{23}$$

with

- α_R: dimensionless exchange strength;
- C: coherence or closure memory;
- H: expansion/activity scale;
- ρ_R: recursive-sector abundance.

The exchange therefore vanishes if any of the following vanish:

$$\alpha_R = 0, \qquad C = 0, \qquad \rho_R = 0. \tag{24}$$

The diagnostic exchange rate per Hubble time per recursive density is

$$\boxed{\frac{\Xi}{H\rho_R} = \alpha_R C}. \tag{25}$$

8.1 Why This Is the Minimal Law

Equation (23) is the lowest-order homogeneous expression that satisfies four requirements:

(i) transfer requires recursive abundance ρ_R;

(ii) transfer requires coherence memory C;

(iii) transfer scales with the cosmological clock H;

(iv) the coupling is dimensionless in e-fold time.

It is not the only possible exchange law. It is the smallest one that keeps the survival-sector interpretation visible while remaining numerically tractable.

8.2 Less-Compressed Exposure Law

If one wants survival exposure to appear explicitly, use

$$\Xi = \alpha_0 C \mathcal{E} H \rho_R, \tag{26}$$

where $\mathcal{E} = \mathrm{d}A/\mathrm{d}N$. The ultra-minimal form is recovered by absorbing the effective exposure into the coupling:

$$\alpha_R \equiv \alpha_0 \mathcal{E}_{\rm eff}. \tag{27}$$

Thus (23) should be read as the lowest-mode homogeneous approximation to survival-mediated exchange, not as the disappearance of the survival machinery.

9 Coherence Memory

The coherence memory C tracks whether recursive histories remain coherently represented enough to affect the ordinary sector. It should decay in the absence of recursive relevance and should not grow without bound. The bounded e-fold-scaled law is

$$\dot{C} = H\left[-\gamma_C C + \eta \Omega_R (1 - C)\right], \tag{28}$$

where

$$\Omega_R = \frac{\rho_R}{\rho + \rho_R}. \tag{29}$$

Equivalently,

$$\boxed{C' = -\gamma_C C + \eta \Omega_R (1 - C)}. \tag{30}$$

Here γ_C and η are dimensionless per e-fold.

9.1 Boundedness

The bounded form is useful because, for

$$\gamma_C \geq 0, \qquad \eta \geq 0, \qquad 0 \leq \Omega_R \leq 1, \tag{31}$$

the interval $0 \leq C \leq 1$ is forward invariant. At $C = 0$,

$$C' = \eta \Omega_R \geq 0, \tag{32}$$

so the flow points inward. At $C = 1$,

$$C' = -\gamma_C \leq 0, \tag{33}$$

so the flow again points inward. This turns C into a genuine memory or saturation variable rather than an unconstrained source.

9.2 Instantaneous Coherence Equilibrium

For fixed Ω_R, the instantaneous equilibrium is

$$C_*(\Omega_R) = \frac{\eta \Omega_R}{\gamma_C + \eta \Omega_R}. \tag{34}$$

Thus $C_* \to 0$ when $\Omega_R \to 0$, and $C_* \to \eta/(\gamma_C + \eta)$ when $\Omega_R \to 1$. This is the intended qualitative behaviour: recursive coherence can be supported while the recursive sector is present, but it fades when that sector disappears.

10 Final E-Fold System

Using $X' = \dot{X}/H$, equations (20), (21), (23), and (30) become

$$\rho' + 3(1+w)\rho = +\alpha_R C \rho_R, \tag{35}$$
$$\rho_R' + 3(1+w_R)\rho_R = -\alpha_R C \rho_R, \tag{36}$$
$$C' = -\gamma_C C + \eta \Omega_R (1-C). \tag{37}$$

Equivalently,

$$\boxed{\rho_R' = -\left[3(1+w_R) + \alpha_R C\right] \rho_R} \tag{38}$$

which makes the recursive depletion channel explicit.

10.1 Reduced System

For many numerical experiments it is cleaner to evolve Ω_R, C, and H. From $\Omega_R = \rho_R/(\rho + \rho_R)$, one obtains

$$\boxed{\Omega_R' = \Omega_R \left[3(w - w_R)(1-\Omega_R) - \alpha_R C\right]}. \tag{39}$$

The coherence equation is

$$\boxed{C' = -\gamma_C C + \eta \Omega_R (1-C)}. \tag{40}$$

The Hubble evolution follows from ρ_{tot} and the effective equation of state:

$$\boxed{\frac{H'}{H} = -\frac{3}{2}\left[1 + w(1-\Omega_R) + w_R \Omega_R\right]}. \tag{41}$$

The effective equation of state is

$$\boxed{w_{\text{eff}}(N) = w(1-\Omega_R) + w_R \Omega_R}. \tag{42}$$

11 Canonical Minimal System

The canonical v0.2 background system is therefore

$$\boxed{\Xi = \alpha_R C H \rho_R,} \tag{43}$$

$$\boxed{C' = -\gamma_C C + \eta \Omega_R (1-C),} \tag{44}$$

$$\boxed{\Omega_R' = \Omega_R \left[3(w - w_R)(1-\Omega_R) - \alpha_R C\right],} \tag{45}$$

$$\boxed{\frac{H'}{H} = -\frac{3}{2}\left[1 + w(1-\Omega_R) + w_R \Omega_R\right].} \tag{46}$$

These equations are the smallest numerical core of the model.

12 Well-Behaved Regime

For the minimal model, take

$$\alpha_R \geq 0, \qquad \gamma_C > 0, \qquad \eta \geq 0, \qquad 0 \leq C_0 \leq 1, \qquad \rho_0, \rho_{R0} \geq 0. \tag{47}$$

Then C remains bounded in $[0, 1]$, and both densities remain non-negative.

The recursive density evolves as

$$\rho_R(N) = \rho_R(N_0) \exp\left[-\int_{N_0}^{N} (3(1 + w_R) + \alpha_R C)\, \mathrm{d}N\right]. \tag{48}$$

Thus ρ_R decays when the integrated bracket is positive.

Late GR recovery requires

$$\Omega_R \to 0, \qquad C \to 0, \qquad \alpha_R C \to 0. \tag{49}$$

12.1 Density Positivity

If $\rho_R(N_0) \geq 0$, equation (48) implies $\rho_R(N) \geq 0$ for all later N. Since the ordinary sector obeys

$$\rho' = -3(1 + w)\rho + \alpha_R C \rho_R, \tag{50}$$

the boundary $\rho = 0$ has

$$\rho'\big|_{\rho=0} = \alpha_R C \rho_R \geq 0 \tag{51}$$

under (47). The ordinary density is therefore also non-negative once non-negative.

12.2 Recursive Fraction

For $0 \leq \Omega_R \leq 1$, equation (39) has boundaries

$$\Omega_R = 0, \qquad \Omega_R = 1. \tag{52}$$

At $\Omega_R = 0$, $\Omega_R' = 0$. At $\Omega_R = 1$,

$$\Omega_R' = -\alpha_R C \leq 0. \tag{53}$$

Thus the interval $0 \leq \Omega_R \leq 1$ is forward invariant for the minimal parameter regime.

13 Parameter Roles and Limiting Cases

13.1 Exchange Strength α_R

The parameter α_R controls the rate at which coherent recursive-sector density is transferred into the ordinary represented sector. It is not itself a density. It is a dimensionless rate per Hubble time once multiplied by C:

$$\frac{\Xi}{H\rho_R} = \alpha_R C. \tag{54}$$

Large α_R rapidly depletes ρ_R, but also risks strong departures from standard expansion. Small α_R may leave too much recursive density or produce a negligible maturation effect.

13.2 Coherence Decay γ_C

The parameter γ_C controls how rapidly coherence memory decays per e-fold. If γ_C is too large, C never becomes dynamically relevant. If it is too small, coherence may persist too long and cause late deviations.

13.3 Coherence Source η

The parameter η controls how efficiently recursive-sector abundance sources coherence memory. It does not source C from nothing; the source is weighted by Ω_R. The factor $(1 - C)$ imposes saturation.

13.4 Recursive Equation of State w_R

The equation-of-state parameter w_R governs the passive redshifting of the recursive sector. Several regimes are especially important:

- If $w_R > w_{\text{late}}$, the recursive sector redshifts away relative to the late ordinary sector.
- If $w_R = w_{\text{late}}$, any leftover Ω_R tends to freeze once C decays, so it must be depleted early.
- If $w_R < w_{\text{late}}$, the recursive sector can become dangerous unless an additional decay or transfer mechanism removes it.

13.5 No-Exchange Limit

If $\alpha_R = 0$, the sectors do not exchange density. The recursive fraction then evolves only through relative redshifting:

$$\Omega_R' = 3(w - w_R)\Omega_R(1 - \Omega_R). \tag{55}$$

This limit is useful as a control case. It tests whether the early effect is really due to recursive exchange rather than simply adding another fluid.

13.6 No-Coherence Limit

If $C = 0$, then $\Xi = 0$. The recursive sector may still affect the expansion through ρ_R, but it cannot feed ordinary represented density. This limit separates "extra background density" from the genuinely recursive transfer mechanism.

13.7 Fast-Coherence Approximation

If C relaxes much faster than Ω_R, one may approximate

$$C \approx C_*(\Omega_R) = \frac{\eta\Omega_R}{\gamma_C + \eta\Omega_R}. \tag{56}$$

Then

$$\Omega_R' \approx \Omega_R \left[3(w - w_R)(1 - \Omega_R) - \alpha_R \frac{\eta\Omega_R}{\gamma_C + \eta\Omega_R}\right]. \tag{57}$$

This reduced one-variable approximation is not the primary model, but it is a useful analytic guide to parameter scans.

14 Transient Early Effect

The model is interesting only if it produces a finite early effect and then switches itself off. The relevant product is not merely C or Ω_R, but

$$C\Omega_R, \tag{58}$$

because this product controls the availability of coherent recursive abundance. The exchange diagnostic is

$$\alpha_R C, \tag{59}$$

while maturation diagnostics will typically depend on $C\Omega_R$ or $C\Omega_R\mathcal{E}$.

A successful run should show:

(i) Ω_R non-negligible only in an early interval;

(ii) C sourced while Ω_R is present;

(iii) $C\Omega_R$ forming a transient bump;

(iv) $\alpha_R C \to 0$ at late times;

(v) $H/H_{\text{standard}} \to 1$ at late times.

15 Toy Maturation Index

This note should not prematurely claim a full perturbation theory for density growth. Instead, introduce a toy maturation index $\mathcal{M}$ whose purpose is only to measure whether coherent recursive abundance can provide a transient enhancement:

$$\boxed{\mathcal{M}' = s_R C\Omega_R \mathcal{M}}. \tag{60}$$

If explicit exposure is retained, use

$$\boxed{\mathcal{M}' = s_R C\Omega_R \mathcal{E}\mathcal{M}}. \tag{61}$$

Here s_R is a phenomenological response coefficient. Integrating (60) gives

$$\frac{\mathcal{M}(N)}{\mathcal{M}(N_0)} = \exp\left[s_R \int_{N_0}^{N} C(\tilde{N})\Omega_R(\tilde{N})\,\mathrm{d}\tilde{N}\right]. \tag{62}$$

Thus the relevant quantity is the area under the $C\Omega_R$ bump. This is more disciplined than inserting an unspecified growth enhancement directly into a matter perturbation equation.

16 First Numerical Protocol

The first numerical experiment should be deliberately modest. Choose a background interval $N \in [N_i, N_f]$, initial values

$$\Omega_R(N_i) = \Omega_{R,i}, \qquad C(N_i) = C_i, \qquad H(N_i) = H_i, \tag{63}$$

and parameters

$$\Theta = \{\alpha_R, \gamma_C, \eta, w, w_R\}. \tag{64}$$

Integrate the reduced system

$$\Omega_R' = \Omega_R \left[3(w - w_R)(1 - \Omega_R) - \alpha_R C\right], \tag{65}$$

$$C' = -\gamma_C C + \eta \Omega_R (1 - C), \tag{66}$$

$$\frac{H'}{H} = -\frac{3}{2}\left[1 + w(1 - \Omega_R) + w_R \Omega_R\right]. \tag{67}$$

16.1 Primary Plots

The first numerical note should plot

$$\Omega_R(N), \qquad C(N), \qquad C\Omega_R, \qquad \alpha_R C, \qquad w_{\text{eff}}(N), \qquad \frac{H(N)}{H_{\text{standard}}(N)}. \tag{68}$$

If a toy maturation index is included, also plot

$$\mathcal{M}(N), \qquad \ln \frac{\mathcal{M}(N)}{\mathcal{M}(N_i)}. \tag{69}$$

16.2 Acceptance Criteria

A parameter point should be considered viable only if:

(i) $0 \le C(N) \le 1$ for the whole integration;

(ii) $0 \le \Omega_R(N) \le 1$ for the whole integration;

(iii) $C\Omega_R$ has a finite early bump;

(iv) $\Omega_R(N_f)$ is below a chosen late-time tolerance;

(v) $\alpha_R C(N_f)$ is below a chosen late-time tolerance;

(vi) H/H_{standard} remains within observationally tolerable bounds;

(vii) the maturation index receives a non-negligible but not arbitrary boost.

16.3 Suggested Scan

A first coarse scan can use logarithmic or semi-logarithmic ranges such as

$$\alpha_R \in [10^{-3}, 10], \qquad \gamma_C \in [10^{-3}, 10], \qquad \eta \in [10^{-3}, 10], \tag{70}$$

with representative choices of w and w_R. The point is not to fit data yet. The point is to identify whether there exists a stable region in which the desired transient behaviour is generic rather than tuned.

17 Interpretation in Recursive-Survival Language

The minimal variables map onto the Recursive-Survival vocabulary as follows:

- Ω^2 shapes the available phase-history flow.
- ΓW tracks survival filtering or loss exposure.
- $A = \int \Gamma W \, d\tau$ tracks accumulated survival loss.
- C tracks coherent recursive memory or closure persistence.
- ρ_R represents unresolved survival-weighted history structure.
- Ξ expresses effective exchange between recursive and ordinary represented sectors.

The homogeneous model compresses this richer structure into the variables ρ_R, C, and α_R. This compression is a modelling choice, not a conceptual erasure. The full Recursive-Survival interpretation remains that represented structure concentrates when some histories preserve coherence, closure, and low-loss propagation better than others.

18 Relation to Known Effective-Fluid Models

At the level of equations, this construction resembles interacting-fluid cosmologies, early dark energy models, and phenomenological decay sectors. That resemblance is a strength rather than a defect: it means the model is entering the same mathematical arena as already-testable effective cosmology.

The difference is interpretive and structural. The recursive sector is not introduced as an arbitrary dark component. It represents unresolved survival-weighted history structure, and its exchange is governed by coherence memory and survival exposure. The model should therefore be judged by both ordinary cosmological constraints and its ability to express the intended RSG mechanism in a disciplined low-dimensional form.

19 Quantum-Gravity-Facing Interpretation

The minimal system above is deliberately conservative: it is written as an effective FLRW model with a conserved recursive sector. However, the same variables can be read in a more quantum-gravity-facing way if the recursive-survival sector is treated less like an ordinary fluid and more like a coarse-grained description of unresolved spacetime histories.

This section does not add new dynamics to the canonical v0.2 model. It gives a possible interpretive bridge. The guiding question is:

> Is the recursive-survival sector merely a useful effective fluid, or is it the visible shadow of unresolved quantum spacetime histories becoming classical?

19.1 Generated Histories and Quantum-History Sums

Many approaches to quantum gravity involve some version of summing over possible fields, geometries, causal structures, or histories. Recursive Survival already speaks about generated histories and survival-weighted representation. A quantum-history-facing version may therefore ask whether recursive histories γ_i should be interpreted as coarse-grained spacetime histories:

$$\text{recursive histories } \gamma_i \quad \text{as coarse-grained quantum spacetime histories.}$$

In the minimal model, survival weight is introduced through

$$S_i = e^{-A_i}.$$

A more explicitly quantum version would compare this with amplitude weighting, decoherence functionals, path-integral suppression, or statistical weights such as

$$p_i \sim \frac{e^{-A_i/\hbar}}{Z}.$$

Here $\hbar$ is restored to mark the quantum scale, and Z is a normalization factor. This does not yet prove that survival weighting is a quantum amplitude. It only identifies the place where such a connection would need to be made.

19.2 Recursive Density as Pre-Classical Geometry

In the minimal FLRW equations, ρ_R is a recursive-sector density. A quantum-gravity-facing reading sharpens this by treating ρ_R as an effective contribution from geometry that has not fully settled into a classical spacetime description:

$$\rho_R \quad \text{as coarse-grained contribution from unresolved quantum geometry.}$$

This interpretation makes the recursive sector a bridge between a microscopic pre-classical layer and the ordinary classical FLRW background. The sector is still constrained by conservation, but its meaning shifts from “extra fluid” to “coarse-grained unresolved geometry/history content.”

19.3 Microscopic Origin of the Recursive Stress-Energy

The present document introduces $T_R^{\mu\nu}$ at the effective level. A more complete quantum-gravity version would try to derive it from microscopic degrees of freedom. For example, one could seek a relation of the schematic form

$$T_R^{\mu\nu} \sim \left\langle \hat{T}^{\mu\nu}_{\text{unresolved}} \right\rangle,$$

where the right-hand side is an expectation value of unresolved microscopic stress-energy. Another possibility is to relate the effective stress-energy to a response of survival action or survival loss to the coarse-grained metric:

$$T_R^{\mu\nu} \sim \frac{\delta A_{\text{survival}}}{\delta g_{\mu\nu}}.$$

Both expressions are only signposts. They say where a microscopic theory would have to enter if the recursive sector is to become more than phenomenology.

19.4 Coherence Memory as Decoherence

The variable C is the most quantum-adjacent ingredient in the minimal system. In the background model it is a bounded coherence or closure memory. In a quantum-history reading, it may be interpreted as the degree of remaining coherence among geometry/history branches:

$$C \quad \text{as coherence between unresolved geometry or history branches.}$$

Then the decay term in

$$C' = -\gamma_C C + \eta \Omega_R (1 - C)$$

resembles a coarse-grained decoherence process. The recursive sector can source coherence while unresolved structure remains relevant, but once Ω_R falls, C decays and the model approaches an effectively classical description.

19.5 Survival Entropy and Gravitational Entropy

Another route toward quantum gravity is entropy. If the recursive sector contains unresolved surviving histories, then one may associate it with a survival entropy $S_{\rm surv}$. A quantum-gravity-facing extension would ask whether this can be compared with gravitational entropy, horizon entropy, or holographic bounds:

$$S_{\rm surv} \longleftrightarrow S_{\rm grav} \longleftrightarrow \frac{A_{\rm hor}}{4G\hbar}.$$

Here $A_{\rm hor}$ denotes a horizon area, not the accumulated survival loss A. This distinction matters because the same letter A would otherwise carry two different meanings.

19.6 Perturbations and Quantum Fluctuations

The present model is a homogeneous background model. Quantum gravity is also concerned with fluctuations and the emergence of classical perturbations from quantum seeds. A stronger version must therefore study perturbations of both the recursive sector and the metric:

$$\delta\rho_R, \qquad \delta C, \qquad \Phi, \qquad \Psi.$$

Here Φ and Ψ are standard scalar metric perturbations. This is where the model could connect to primordial structure, quantum fluctuations, and the emergence of classical density seeds. The current toy maturation index is not a substitute for this perturbation theory; it is only a first diagnostic.

19.7 Restoring $\hbar$

The canonical v0.2 equations set $c = 1$ and do not use $\hbar$. That is appropriate for a classical effective-fluid scaffold. A quantum-gravity-facing extension should restore $\hbar$ wherever survival weighting is compared to a quantum action, amplitude, or statistical path weight:

$$S_i \sim e^{-A_i/\hbar}, \qquad p_i \sim \frac{e^{-A_i/\hbar}}{Z}.$$

This makes clear whether survival filtering is intended as genuinely quantum-like or merely mathematically similar to statistical weighting.

19.8 Conservative Summary

The minimal model can remain exactly what it is: a conserved effective sector for early FLRW maturation. The quantum-gravity-facing extension asks whether that sector is the coarse-grained remnant of unresolved quantum spacetime histories. A possible title for that next-stage version is:

Recursive Survival as a Coarse-Grained Quantum-History Sector in Early Cosmology.

This title keeps the current model intact while aiming the interpretation more clearly toward the emergence of classical cosmology from unresolved quantum history structure.

20 Falsifiability

The model fails if any of the following hold:

- ρ_R remains too large at late times.

- C does not decay or saturate naturally.
- The expansion history departs too strongly from standard constraints.
- The maturation boost is negligible for all allowed parameters.
- The required parameters must be tuned separately for each observation.
- The model cannot produce a transient $C\Omega_R$ window without leaving a late remnant.

The model gains credibility only if a compact parameter range gives early maturation, late GR recovery, and observationally acceptable expansion.

21 Next Extensions

The minimal model is intentionally small. Natural next extensions include:

(i) splitting the ordinary sector into radiation, baryons, cold dark matter, and vacuum density;

(ii) allowing $\mathcal{E}(N)$ to vary explicitly rather than being absorbed into α_R;

(iii) adding a perturbation-level treatment for the recursive sector;

(iv) allowing momentum exchange in Q^ν outside exact FLRW;

(v) constraining H/H_{standard} against early-universe and late-time observables;

(vi) testing whether the $C\Omega_R$ bump can help with early structural maturation without mimicking an already-excluded early dark component.

(vii) interpreting ρ_R as coarse-grained unresolved quantum geometry and testing whether $T_R^{\mu\nu}$ can be derived from microscopic or path-integral degrees of freedom.

(viii) connecting coherence memory C to decoherence between quantum geometry/history branches.

(ix) comparing survival entropy with gravitational entropy, horizon entropy, and holographic bounds.

22 Closing Statement

The minimal Recursive-Survival cosmology is not "Einstein plus mystery." It is a two-sector conserved system in which a transient recursive-survival fluid can exchange with ordinary represented structure through coherence and survival exposure. Its purpose is to test whether early structural maturity can arise from survival-weighted concentration rather than infinite processing or unconstrained new physics.

A Derivation of the Reduced Ω_R Equation

Let

$$\Omega_R = \frac{\rho_R}{\rho + \rho_R} = \frac{\rho_R}{\rho_{\text{tot}}}. \tag{71}$$

Then

$$\frac{\Omega_R'}{\Omega_R} = \frac{\rho_R'}{\rho_R} - \frac{\rho_{\text{tot}}'}{\rho_{\text{tot}}}. \tag{72}$$

From the recursive continuity equation,

$$\frac{\rho_R'}{\rho_R} = -3(1 + w_R) - \alpha_R C. \tag{73}$$

The total density obeys

$$\rho_{\text{tot}}' = -3(1 + w)\rho - 3(1 + w_R)\rho_R. \tag{74}$$

Since

$$\rho = (1 - \Omega_R)\rho_{\text{tot}}, \qquad \rho_R = \Omega_R \rho_{\text{tot}}, \tag{75}$$

we get

$$\frac{\rho_{\text{tot}}'}{\rho_{\text{tot}}} = -3\left[(1 + w)(1 - \Omega_R) + (1 + w_R)\Omega_R\right]. \tag{76}$$

Therefore

$$\frac{\Omega_R'}{\Omega_R} = -3(1 + w_R) - \alpha_R C + 3\left[(1 + w)(1 - \Omega_R) + (1 + w_R)\Omega_R\right] \tag{77}$$

$$= 3(w - w_R)(1 - \Omega_R) - \alpha_R C. \tag{78}$$

Thus

$$\Omega_R' = \Omega_R\left[3(w - w_R)(1 - \Omega_R) - \alpha_R C\right]. \tag{79}$$

B Derivation of the Hubble Equation

For a flat model,

$$H^2 = \frac{8\pi G}{3}\rho_{\text{tot}}. \tag{80}$$

Taking a logarithmic derivative with respect to N,

$$2\frac{H'}{H} = \frac{\rho_{\text{tot}}'}{\rho_{\text{tot}}}. \tag{81}$$

Using

$$\frac{\rho_{\text{tot}}'}{\rho_{\text{tot}}} = -3(1 + w_{\text{eff}}), \tag{82}$$

with

$$w_{\text{eff}} = w(1 - \Omega_R) + w_R \Omega_R, \tag{83}$$

gives

$$\frac{H'}{H} = -\frac{3}{2}\left[1 + w(1 - \Omega_R) + w_R \Omega_R\right]. \tag{84}$$

C Dimensional Bookkeeping

With $c = 1$, ρ and ρ_R have dimensions of energy density, while H, ΓW, and Ξ/ρ_R have dimensions of inverse time. The exchange term

$$\Xi = \alpha_R C H \rho_R \tag{85}$$

has dimensions

$$[\Xi] = [H][\rho_R], \tag{86}$$

as required for a term in $\dot{\rho}$. The variables α_R, C, γ_C, η, Ω_R, and $\mathcal{E}$ are dimensionless.

D Minimal Pseudocode

A first implementation can be written schematically as follows:

```
choose parameters alpha_R, gamma_C, eta, w, w_R
choose initial values N_i, N_f, Omega_R_i, C_i, H_i

for N from N_i to N_f:
    Omega_R_prime = Omega_R * (
        3*(w - w_R)*(1 - Omega_R) - alpha_R*C
    )
    C_prime = -gamma_C*C + eta*Omega_R*(1 - C)
    H_prime_over_H = -1.5 * (
        1 + w*(1 - Omega_R) + w_R*Omega_R
    )
    integrate Omega_R, C, ln(H)
    record C*Omega_R, alpha_R*C, w_eff, H/H_standard
```

E Compact Summary

The whole minimal model can be compressed to the following statement:

> Add one conserved recursive-survival fluid to FLRW. Let it exchange density with the ordinary represented sector through $\Xi = \alpha_R C H \rho_R$. Let coherence obey the bounded memory law $C' = -\gamma_C C + \eta \Omega_R (1 - C)$. Test whether $C\Omega_R$ develops a transient early bump while Ω_R, C, and $\alpha_R C$ all vanish at late times.