Entropy in Recursive Survival Geometry
Shannon Readout and Survival-Closure Update

Draft note, Peter M Austin

18 May 2026

Abstract

In Recursive Survival Geometry, entropy is best introduced not as primitive disorder, but as the logarithmic measure of unresolved, survival-compatible history space. Recursive dynamics generate candidate histories. Survival filtering then assigns each history a persistence weight. After normalisation, these weights define the represented measure. Shannon entropy is the scalar readout of that normalised survival measure: it measures the uncertainty, multiplicity, or effective number of histories that remain live under representation. This update also records the entropy-flow identity and the survival-closure diagnostic derived from the represented measure. No new empirical claim is added to the main RSG formalism.

1 Core statement

Principle 1 (Entropy as unresolved surviving history)**.** *Entropy is the logarithmic measure of unresolved, survival-compatible histories. Equivalently, it is the effective number of distinguishable histories that remain live after recursive generation and survival filtering.*

Thus entropy is not first defined as disorder, heat, or randomness. Those are later coarse-grained appearances. The primitive RSG definition is measure-theoretic:

$$\boxed{\text{entropy} = \text{uncertainty in the normalised survival measure over histories.}}$$

2 Placement of the Shannon update

The entropy document already defines entropy as the logarithmic measure of unresolved, survival-compatible history space. The present update makes the measurement step explicit. It should be read as a definitional refinement:

$$\text{generated histories} \longrightarrow \text{survival weights} \longrightarrow \text{normalised representation measure} \longrightarrow \text{Shannon scalar.}$$

No new empirical claim is added. The main text continues to hold the formal and analogue-test claims separately. This note only clarifies how entropy is computed once the represented measure has already been defined.

3 Recursive histories and survival weights

Let a structured recursive state be written

$$\sigma_n = (X_n, \phi_n, \mu_n, S_n),$$

where X_n is the topological support, ϕ_n is phase or transport data, μ_n stores physical measure data, and S_n is the survival weight. A generated history is a sequence

$$\gamma_i = (\sigma_{i,0}, \sigma_{i,1}, \sigma_{i,2}, \ldots).$$

The recursive update is

$$\sigma_{n+1} = R(\sigma_n).$$

When a projected phase description is used, write

$$\phi_n = (\Theta_n, \Pi_n).$$

In a continuous projected approximation,

$$\frac{\mathrm{d}\Theta}{\mathrm{d}t} = \Pi, \qquad \frac{\mathrm{d}\Pi}{\mathrm{d}t} = -\Omega^2\Theta.$$

For a family of candidate histories $\{\gamma_i\}$, assign each history a survival weight $S_i(t) \geq 0$. The represented measure is the normalised survival measure

$$\boxed{p_i(t) = \frac{S_i(t)}{\sum_j S_j(t)}.}$$

This p_i is not the raw probability that a history was generated. It is the probability with which a history remains represented after filtering.

In continuous form, a common survival law is

$$\frac{dS_i}{dt} = -\Gamma_i(\sigma)W_i(\phi)S_i,$$

where $\Gamma_i \geq 0$ is a local loss coefficient and W_i is exposure to the loss channel. Using the local action norm

$$J(\phi) = \Theta^2 + \ell^2\Pi^2,$$

a typical exposure factor is

$$W(\phi) = \frac{\Theta^2}{J(\phi)}.$$

Define the effective survival-loss rate

$$\lambda_i(t) = \Gamma_i(\sigma_i(t))W_i(\phi_i(t)).$$

Equivalently,

$$S_i(t) = \exp[-A_i(t)], \qquad A_i(t) = \int_0^t \Gamma_i(\tau)W_i(\tau)\,d\tau = \int_0^t \lambda_i(\tau)\,\mathrm{d}\tau,$$

up to a common initial normalisation.

Then

$$p_i(t) = \frac{e^{-A_i(t)}}{Z(t)}, \qquad Z(t) = \sum_j e^{-A_j(t)}.$$

This makes survival filtering formally resemble a Gibbs weighting, with the accumulated loss action A_i playing the role of an effective cost.

4 Survival-representation entropy

Definition 1 (Survival entropy). *The survival-representation entropy is*

$$\boxed{\mathcal{H}_{\mathrm{surv}}(t) = -\sum_i p_i(t) \ln p_i(t).}$$

For a represented history i, the Shannon information content in base b is

$$I_b(i) = -\log_b p_i.$$

The average information content of the represented survival measure is

$$H_b(p) = \sum_i p_i I_b(i) = -\sum_i p_i \log_b p_i.$$

The base b only fixes units. Base 2 gives bits; base e gives nats. Since the RSG survival weights are written with e^{-A_i}, the natural convention in this note is the natural-logarithm form $\mathcal{H}_{\mathrm{surv}}$ above.

Using $p_i = e^{-A_i}/Z$, survival entropy can also be written

$$\mathcal{H}_{\mathrm{surv}}(t) = \langle A \rangle_p + \ln Z,$$

where

$$\langle A \rangle_p = \sum_i p_i A_i.$$

The associated effective number of surviving histories is

$$\boxed{N_{\mathrm{eff}}(t) = e^{\mathcal{H}_{\mathrm{surv}}(t)}.}$$

Thus:

$$\mathcal{H}_{\mathrm{surv}} \text{ high} \quad \Longleftrightarrow \quad \text{many distinguishable histories remain live,}$$

while

$$\mathcal{H}_{\mathrm{surv}} \text{ low} \quad \Longleftrightarrow \quad \text{survival has concentrated representation into a narrow basin.}$$

Shannon entropy is therefore not imported as primitive disorder. It is the scalar readout of the normalised survival distribution.

In operator language, the informational layer may be written schematically as

$$d \xrightarrow{n} p \xrightarrow{\mathcal{S}_{\mathrm{Sh}}} \mathcal{H}_{\mathrm{surv}} \xrightarrow{s} \text{scalar readout,}$$

or compactly,

$$K_{\mathrm{info}} = s \circ \mathcal{S}_{\mathrm{Sh}} \circ n \circ d, \qquad K^*_{\mathrm{info}} \sim s \cdot \mathcal{S}_{\mathrm{Sh}} \cdot n \cdot d.$$

5 Entropy flow under survival filtering

The survival measure evolves according to

$$\dot{p}_i = p_i\big(\langle \lambda \rangle_p - \lambda_i\big), \qquad \langle \lambda \rangle_p = \sum_j p_j \lambda_j.$$

Therefore a history gains represented weight exactly when its current effective loss is below the represented average:

$$\lambda_i < \langle \lambda \rangle_p \quad \Longrightarrow \quad \dot{p}_i > 0.$$

Using

$$\mathcal{H}_{\text{surv}} = \langle A \rangle_p + \ln Z,$$

one obtains the concise entropy-flow identity

$$\dot{\mathcal{H}_{\text{surv}}} = -\operatorname{Cov}_p(A, \lambda).$$

Equivalently, since $\lambda = \Gamma W$ along a history,

$$\dot{\mathcal{H}_{\text{surv}}} = -\operatorname{Cov}_p(A, \Gamma W).$$

If histories that have already accumulated high loss also continue to have high current loss, then $\operatorname{Cov}_p(A, \lambda) > 0$, and represented entropy falls:

$$\dot{\mathcal{H}_{\text{surv}}} < 0.$$

The represented measure is concentrating into a narrower survival basin. If all histories share the same effective loss, $\lambda_i = \lambda$, then

$$\dot{p}_i = 0, \qquad \dot{\mathcal{H}_{\text{surv}}} = 0,$$

so the survival filter introduces no additional representation contrast.

6 Local concentration rule

Recursive survival geometry separates generation from selection. The phase-flow coefficient Ω^2 shapes which histories are dynamically accessible in the projected phase portrait. The loss factor ΓW then selects among those histories by differential persistence. Shannon entropy measures the multiplicity that remains after this two-stage process.

For a region R of the projected history space, define the represented density

$$\rho_R(t) = \sum_{i:\gamma_i(t)\in R} p_i(t).$$

If the membership of R is fixed during the comparison interval, then

$$\dot{\rho}_R = \rho_R\big(\langle \lambda \rangle_p - \langle \lambda \rangle_R\big),$$

where

$$\langle \lambda \rangle_R = \frac{1}{\rho_R} \sum_{i:\gamma_i(t)\in R} p_i \lambda_i.$$

Hence

$$\dot{\rho}_R > 0 \quad \Longleftrightarrow \quad \langle \Gamma W \rangle_R < \langle \Gamma W \rangle_p.$$

This is the local concentration rule in entropy language: regions of lower effective loss gain represented density. Entropy decreases in the represented sector when the survival filter resolves many live possibilities into fewer dominant basins.

7 Generated entropy, represented entropy, and exported entropy

It is useful to distinguish three quantities:

$$\mathcal{H}_{\text{gen}}, \qquad \mathcal{H}_{\text{surv}}, \qquad S_{\text{env}}.$$

Here $\mathcal{H}_{\text{gen}}$ measures the multiplicity of generated candidate histories before selection, $\mathcal{H}_{\text{surv}}$ measures the multiplicity of represented histories after survival filtering, and S_{env} denotes entropy exported to unresolved environmental, loss, or discarded degrees of freedom.

A local concentration process can therefore satisfy

$$\mathcal{H}_{\text{surv}} \downarrow$$

while

$$S_{\text{env}} \uparrow .$$

This is the key distinction. Survival filtering may reduce the visible or represented entropy without violating a thermodynamic second-law interpretation, because the lost or unresolved channel can carry the compensating entropy.

8 Open and closed survival regimes

For a finite active family of N histories,

$$H_{\max} = \ln N.$$

A convenient dimensionless openness index is

$$O_{\text{surv}} = \frac{\mathcal{H}_{\text{surv}}}{H_{\max}},$$

and the complementary survival-closure index is

$$C_{\text{surv}} = 1 - \frac{\mathcal{H}_{\text{surv}}}{H_{\max}}.$$

Thus

$$O_{\text{surv}} \approx 1 \quad \text{means many histories remain unresolved,}$$

while

$$C_{\text{surv}} \approx 1 \quad \text{means representation has concentrated into a narrow basin.}$$

This does not make entropy identical with closure. Rather, entropy is the scalar diagnostic from which a closure index can be derived after the active history family or coarse-graining has been fixed.

9 Topological or closure-class entropy

The support component X_n can be analysed using a partition topology. Let

$$\text{int}_D(X), \qquad \text{cl}_D(X), \qquad \text{bd}_D(X), \qquad \text{class}_D(X)$$

denote the Surtea-style interior, closure, boundary, and class diagnostics.

Instead of resolving individual histories, one may coarse-grain them by topological class. For a class C, define

$$p_C(t) = \sum_{i:\, \text{class}_D(X_i)=C} p_i(t).$$

Definition 2 (Closure-class entropy). *The closure-class entropy is*

$$\boxed{\mathcal{H}_{\text{class}}(t) = -\sum_C p_C(t) \ln p_C(t).}$$

This measures how many distinct closure, boundary, or objecthood classes remain live after filtering. Individual histories may differ while still belonging to the same topological class; $\mathcal{H}_{\text{class}}$ forgets those within-class distinctions.

A useful discrete survival update with topological loss is

$$S_{n+1} = S_n \exp[-L_D(\sigma_n, \sigma_{n+1})\Delta t],$$

with

$$L_D(\sigma_n, \sigma_{n+1}) = \lambda W + \alpha \Delta_{\text{bd}} + \beta \Delta_{\text{class}} + \chi \Delta_{\text{int}}.$$

Here Δ_{bd} measures boundary change, Δ_{class} measures class change, and Δ_{int} measures interaction or closure instability.

10 Thermodynamic entropy as coarse-grained survival entropy

Let M denote a macroscopic state or observational bin. The coarse-grained probability of M is

$$p_M(t) = \sum_{i:\,\gamma_i \in M} p_i(t).$$

Then the coarse-grained entropy is

$$\mathcal{H}_{\text{coarse}}(t) = -\sum_M p_M(t) \ln p_M(t).$$

The physical thermodynamic entropy is then written

$$\boxed{S_{\text{phys}}(t) = k_B \mathcal{H}_{\text{coarse}}(t).}$$

In this reading, thermodynamic entropy is not primitive. It is the macroscopic appearance of survival entropy after microscopic recursive distinctions have been ignored.

11 Continuous history spaces

For an infinite or continuous family of histories, a base history measure must be fixed before entropy is meaningful. With base measure $\mathrm{d}\mu(\gamma)$,

$$\mathrm{d}P(\gamma) = \frac{e^{-A[\gamma]}}{Z}\,\mathrm{d}\mu(\gamma), \qquad Z = \int e^{-A[\gamma]}\,\mathrm{d}\mu(\gamma).$$

The normalisability condition is

$$0 < Z < \infty.$$

The corresponding density relative to the locked base measure is

$$p(\gamma) = \frac{\mathrm{d}P}{\mathrm{d}\mu} = \frac{e^{-A[\gamma]}}{Z},$$

and the entropy readout is

$$\mathcal{H}^{\mu}_{\text{surv}} = -\int p(\gamma) \ln p(\gamma)\,\mathrm{d}\mu(\gamma).$$

Because continuous entropy depends on the chosen base measure, this quantity must not be treated as coordinate-free until the history measure and coarse-graining have been fixed. The safe RSG statement is therefore:

> Entropy is the Shannon readout of the represented survival measure, relative to a specified history family, base measure, and coarse-graining.

12 Light-like and matter-like regimes

In RSG language, a light-like regime is associated with lossless, norm-preserving, non-closing transport. Schematically,

$$\Gamma \to 0, \qquad \|\sigma_{n+1}\|_\Phi = \|\sigma_n\|_\Phi, \qquad r \notin \mathbb{Q}.$$

In this limit, survival entropy is not primarily reduced by dissipation. The transport remains represented without settling into a recurrent rest-frame-like closure.

Matter-like concentration belongs to the complementary regime in which survival loss varies across histories:

$$\Gamma_i W_i \neq \Gamma_j W_j.$$

After normalisation, lower-loss histories gain representation relative to higher-loss histories. Thus clumping is described as measure concentration:

$$p_i(t) = \frac{e^{-A_i(t)}}{\sum_j e^{-A_j(t)}}.$$

Matter-like structure is therefore not introduced as primitive attraction. It is represented as the dominance of histories that persist under the recursive filter.

13 Geometric and entanglement bridge

A cautious bridge to gravitational entropy can be stated as follows. In a continuum or geometric limit, field equations may correspond to stationarity of an appropriate entropy functional under fixed boundary, volume, closure, or support constraints.

A schematic form is

$$\delta\left(\mathcal{H}_{\text{surv}} + \eta A_{\partial X}\right)\Big|_{V,\,\text{class},\,\mu} = 0,$$

where $A_{\partial X}$ is a boundary-area term and η is an area-density coefficient. This is not yet a derivation of Einstein's equation. It is a bridge principle: geometry may be the continuum condition under which survival, boundary, and entanglement entropy are stationary together.

14 Summary

The preferred RSG definition is

$$\boxed{\mathcal{H}_{\text{surv}} = -\sum_i p_i \ln p_i, \qquad p_i = \frac{S_i}{\sum_j S_j}.}$$

Equivalently, when $p_i = e^{-A_i}/Z$,

$$\boxed{\mathcal{H}_{\text{surv}} = \langle A\rangle_p + \ln Z, \qquad \dot{\mathcal{H}}_{\text{surv}} = -\operatorname{Cov}_p(A, \Gamma W).}$$

The preferred plain-language formulation is:

Entropy is the effective number of unresolved histories that still survive as live possibilities.

High entropy means many survival-compatible histories or classes remain live. Low entropy means recursive filtering has concentrated representation into a narrower coherent basin. Thermodynamic entropy is the coarse-grained physical form of this measure. Topological entropy is its closure-class form. Geometric or entanglement entropy is the continuum bridge form.

Acknowledgements

Initial acknowledgements are due to the Information Physics Institute faculty and correspondence community for the shared context in which this entropy note is being developed, including Boris Kreiger and Jeff Dillon. Thanks are also due to the broader RSG and IPI correspondence circle whose questions helped sharpen the distinction between generated histories, represented survival measure, Shannon readout, and empirical claim status.