Universe: A Topological Theory

Version 1 (English translation)

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> *"For where he reaches there is no boundary,*
> *Nor any eye by which to know it,*
> *And time tries in vain*
> *To be born from voids."*
>
> M. Eminescu, *Luceafărul*

Translator's Note

This is an English translation of Traian Surtea's Romanian manuscript *Univers: o teorie topologică v1.*

The author introduces a number of technical neologisms, including *$\mathcal{D}$-photon*, *etheron*, *gluon*, *spation*, *tempon*, *korpuskon*, *undon*, *lighton*, *quantonic*, and *broglionic.* These terms have been preserved in translation, because they function as formal labels inside the author's theory. Where necessary, ordinary English explanation is added around them, but the labels themselves are not replaced by conventional physics terms.

The mathematical notation has been lightly standardised for readability and then updated according to the author's corrections. In particular, collections of subsets are written with calligraphic letters, such as $\mathcal{D}$, $\mathcal{T}$, $\mathcal{S}$, $\mathcal{K}$, and $\mathcal{W}$; structured sets such as the universe and sub-universes are written with blackboard-bold letters, such as $\mathbb{U}$ and $\mathbb{V}$; and lattices are written with fraktur letters, such as $\mathfrak{Par}(M)$ and $\mathfrak{Top}(M)$. The interior projection is written as $\mathring{\pi}_{\mathcal{D}}$, following the author's correction.

Abstract

This is only Chapter 2 of a larger work, *The Theory of Generalised Potential*, but it is self-contained. The connection between this article and TPG will be explained there.

The aim is to define the "Universe", "physical objects", and "interactions" mathematically, starting from very simple notions. These are not yet measurable notions. Those are the subject of Chapter 3 of TPG.

You will not find black holes, the Big Bang, or dark matter here, but neither will you find sophisticated mathematical constructions that would make this material unreadable.

Contents

1 "Specifying" the Universe

1.1 Definition of the Universe

Definition 1.1. *A Universe is the mathematical object*

$$\mathbb{U} = (M, \mathcal{D})$$

where M is a non-empty set, with at least two points, and $\mathcal{D}$ is a partition of the set M. Thus

$$M \neq \varnothing, \qquad \operatorname{card}(M) > 2, \qquad \mathcal{D} \in \mathfrak{Par}(M).$$

Observation. *If M has only one element, the theory becomes trivial.*

The elements of the set M may be regarded as "material points". The components of the partition $\mathcal{D}$ are non-empty subsets of M, pairwise disjoint, whose union is M. They will be denoted by D and will be called $\mathcal{D}$-photons.

If one wants a sub-Universe, then it is

$$\mathbb{V} = (V, \mathcal{V}),$$

where V is a non-empty subset of M, and $\mathcal{V}$ is obtained from $\mathcal{D}$ by taking all non-empty intersections

$$D \cap V, \qquad D \in \mathcal{D}.$$

In addition, if

$$\{a\} \in \mathcal{D},$$

then the $\mathcal{D}$-photon $\{a\}$, consisting of a single "material point", will be called an absolute etheron.

A $\mathcal{D}$-photon

$$G = \{g_1, g_2\} \in \mathcal{D}$$

will be called an absolute gluon.

1.2 Construction of the Topology

We attach a topology to the partition $\mathcal{D}$.

Note. *For every*

$$\varnothing \subseteq \mathcal{X} \subseteq \mathcal{D}$$

we use the notation

$$\cup[\mathcal{X}] := \bigcup_{D \in \mathcal{X}} D = \{x \in M \mid \exists D \in \mathcal{X} \text{ such that } x \in D\},$$

in order to simplify the notation. It simply means the union of subsets.

The usual convention is

$$\cup[\varnothing] = \varnothing.$$

We also have

$$\cup[\mathcal{D}] = M.$$

Definition 1.2. *We define*

$$[\mathcal{D}] = \{\cup[\mathcal{K}] \mid \varnothing \subseteq \mathcal{K} \subseteq \mathcal{D}\}.$$

Observation. *The family* $[\mathcal{D}]$ *is formed from all subsets of* M *obtained by taking the union of some of the subsets* D *from* $\mathcal{D}$*.*

It is important that each $U \in [\mathcal{D}]$ *can be described as*

$$U = \cup[\mathcal{U}]$$

for some

$$\varnothing \subseteq \mathcal{U} \subseteq \mathcal{D}.$$

For professionals, it is easy to see that $[\mathcal{D}]$ *is a topology with basis* $\mathcal{D}$*, but we prove the following proposition.*

Proposition 1.1. $[\mathcal{D}]$ *is a closed-open topology on* M*.*

Proof. First,

$$\varnothing = \cup[\varnothing] \in [\mathcal{D}]$$

and

$$M = \cup[\mathcal{D}] \in [\mathcal{D}].$$

Second, let

$$U_i \in [\mathcal{D}], \qquad i \in I,$$

and let

$$U = \bigcup_{i \in I} U_i.$$

For every $i \in I$, we have

$$U_i = \cup[\mathcal{U}_i]$$

for some

$$\varnothing \subseteq \mathcal{U}_i \subseteq \mathcal{D}.$$

Let

$$\mathcal{U} = \bigcup_{i \in I} \mathcal{U}_i.$$

Then

$$U = \cup[\mathcal{U}] \in [\mathcal{D}].$$

Third, let

$$U_1 \in [\mathcal{D}], \qquad U_2 \in [\mathcal{D}],$$

and let

$$U = U_1 \cap U_2.$$

Since

$$U_1 = \cup[\mathcal{U}_1] \qquad \text{and} \qquad U_2 = \cup[\mathcal{U}_2]$$

for some

$$\varnothing \subseteq \mathcal{U}_1, \mathcal{U}_2 \subseteq \mathcal{D},$$

we have

$$U = \cup[\mathcal{U}_1] \cap \cup[\mathcal{U}_2] = \cup[\mathcal{U}_1 \cap \mathcal{U}_2] \in [\mathcal{D}].$$

Therefore $[\mathcal{D}]$ is a topology on M.

Fourth, let

$$U = \cup[\mathcal{U}] \in [\mathcal{D}]$$

and let

$$\mathcal{V} = \{D \in \mathcal{D} \mid D \cap U = \varnothing\}.$$

Then

$$V = \cup[\mathcal{V}] \in [\mathcal{D}].$$

Simply put, V is the union of the other $D \in \mathcal{D}$ which do not participate in the definition of U. Since

$$U \cup V = M, \qquad U \cap V = \varnothing, \qquad V \in [\mathcal{D}],$$

it follows that U is closed in $[\mathcal{D}]$.

Thus every element of $[\mathcal{D}]$ is both open and closed. The proposition is proved. □

Observation. *The object*

$$([\mathcal{D}], \cup, \cap, \complement)$$

is also a special Boolean algebra. The operations $\cup$ *and* $\cap$ *may be performed for an arbitrary family of elements, and the elements*

$$D \in \mathcal{D}$$

are atoms.

Also, each $D \in \mathcal{D}$ *is the smallest* $[\mathcal{D}]$*-open subset with respect to inclusion, and is therefore a connected component.*

1.3 Interior and Closure Projections

The interior and closure topological projections are defined in the classical way: the interior of a subset is the largest open set, with respect to inclusion, contained in that subset, and the closure is the smallest closed set, with respect to inclusion, which contains that subset. We recall the definitions.

Definition 1.3 (Original numbering: 1.2.2)**.** *Let* $X \in \mathcal{P}(M)$*. We define*

$$\mathring{\pi}_{\mathcal{D}}(X) = X^{\circ},$$

where

$$X^{\circ} \in [\mathcal{D}], \qquad X^{\circ} \subseteq X,$$

and

$$\forall U \in [\mathcal{D}], \quad U \subseteq X \Longrightarrow U \subseteq X^{\circ}.$$

Thus X° *is the largest* $[\mathcal{D}]$*-open subset contained in* X*.*

We also define

$$\overline{\pi}_{\mathcal{D}}(X) = \overline{X},$$

where

$$\overline{X} \in [\mathcal{D}], \qquad X \subseteq \overline{X},$$

and

$$\forall V \in [\mathcal{D}], \quad X \subseteq V \Longrightarrow \overline{X} \subseteq V.$$

Thus $\overline{X}$ *is the smallest* $[\mathcal{D}]$*-closed subset containing* X*.*

Observation. *In the definition of closure, we use* $V \in [\mathcal{D}]$*, because by Proposition 1.2.1 all sets in* $[\mathcal{D}]$ *are at the same time both closed and open.*

Until Section 3, we use X° *for the interior and* $\overline{X}$ *for the closure, in order to simplify the notation.*

The fact that

$$\mathring{\pi}_{\mathcal{D}} : \mathcal{P}(M) \to [\mathcal{D}] \subseteq \mathcal{P}(M)$$

and

$$\overline{\pi}_{\mathcal{D}} : \mathcal{P}(M) \to [\mathcal{D}] \subseteq \mathcal{P}(M)$$

are projections, in the sense that

$$\pi \circ \pi = \pi,$$

is well known.

It is also evident that, for every $X \in \mathcal{P}(M)$*,*

$$\varnothing \subseteq X^\circ \subseteq X \subseteq \overline{X} \subseteq M.$$

Lemma 1.1 (Original numbering: 1.2.1)**.** *For every* $X \in \mathcal{P}(M)$*,*

$$X^\circ = X \iff X = \overline{X} \iff X^\circ = \overline{X}.$$

Proof. If $X^\circ = X$, then since $X^\circ \in [\mathcal{D}]$, and since $[\mathcal{D}]$ is a closed-open topology, we have

$$X^\circ = \overline{X}.$$

Therefore

$$X = \overline{X}.$$

Conversely, if $X = \overline{X}$, then, since $\overline{X} \in [\mathcal{D}]$, we have $X \in [\mathcal{D}]$, and so

$$X^\circ = X.$$

Finally, if $X^\circ = \overline{X}$, then from the inclusions

$$X^\circ \subseteq X \qquad \text{and} \qquad X \subseteq \overline{X},$$

we immediately obtain

$$X^\circ = X, \qquad X = \overline{X}.$$

The lemma is proved. □

Lemma 1.2 (Original numbering: 1.2.2)**.** *For all* $A, B \in \mathcal{P}(M)$*, we have*

$$A \subseteq B \Longrightarrow \mathring{\pi}_{\mathcal{D}}(A) \subseteq \mathring{\pi}_{\mathcal{D}}(B),$$

and

$$A \subseteq B \Longrightarrow \overline{\pi}_{\mathcal{D}}(A) \subseteq \overline{\pi}_{\mathcal{D}}(B).$$

Observation. *This means that the topological operator* $\mathring{\pi}_{\mathcal{D}}$ *is monotonically increasing and contractive. That is, for every* $X \in \mathcal{P}(M)$*,*

$$\mathring{\pi}_{\mathcal{D}}(X) \subseteq X.$$

The topological operator $\overline{\pi}_{\mathcal{D}}$ *is monotonically increasing and expansive. That is, for every* $X \in \mathcal{P}(M)$*,*

$$X \subseteq \overline{\pi}_{\mathcal{D}}(X).$$

1.4 Topological Classification of the Subsets of M

The criterion of classification is based on the inclusions

$$\varnothing \subseteq X^\circ \subseteq X \subseteq \overline{X} \subseteq M,$$

where X° is the interior of the subset X, and $\overline{X}$ is the closure of the same subset with respect to the closed-open topology under consideration.

Because we have the equivalences

$$\overline{X} = X \iff X^\circ = X \iff X^\circ = \overline{X},$$

some of the equality or inequality cases in the following table are impossible.

Hex	Binary	Structure	Topological character	Name
0	0000	$\varnothing \neq X^\circ \neq X \neq \overline{X} \neq M$	neither sparse nor dense	$\mathcal{D}$-korpuskons
1	0001	$\varnothing \neq X^\circ \neq X \neq \overline{X} = M$	dense, not sparse	$\mathcal{D}$-tempons
2	0010	$\varnothing \neq X^\circ \neq X = \overline{X} \neq M$	impossible	
3	0011	$\varnothing \neq X^\circ \neq X = \overline{X} = M$	impossible	
4	0100	$\varnothing \neq X^\circ = X \neq \overline{X} \neq M$	impossible	
5	0101	$\varnothing \neq X^\circ = X \neq \overline{X} = M$	impossible	
6	0110	$\varnothing \neq X^\circ = X = \overline{X} \neq M$	$X \in [\mathcal{D}]$, not sparse, not dense	$\mathcal{D}$-lightons
7	0111	$\varnothing \neq X^\circ = X = \overline{X} = M$	$X = M$	"the total lighton"
8	1000	$\varnothing = X^\circ \neq X \neq \overline{X} \neq M$	sparse, not dense	$\mathcal{D}$-spations
9	1001	$\varnothing = X^\circ \neq X \neq \overline{X} = M$	both sparse and dense	$\mathcal{D}$-undons
A	1010	$\varnothing = X^\circ \neq X = \overline{X} \neq M$	impossible	
B	1011	$\varnothing = X^\circ \neq X = \overline{X} = M$	impossible	
C	1100	$\varnothing = X^\circ = X \neq \overline{X} \neq M$	impossible	
D	1101	$\varnothing = X^\circ = X \neq \overline{X} = M$	impossible	
E	1110	$\varnothing = X^\circ = X = \overline{X} \neq M$	$\varnothing = X$	"the null lighton"
F	1111	$\varnothing = X^\circ = X = \overline{X} = M$	impossible	

The author apologises for these barbarisms, but they were necessary in order to avoid confusion with the usual meanings of the words "light", "particle", "space", "time", and "wave".

Definition 1.4 (Original numbering: 1.2.3). *We write*

$$\mathcal{P}_{\mathcal{D}}^{\text{phys}} = \{X \subseteq M \mid \varnothing \neq X \neq M,\ X \notin [\mathcal{D}]\}.$$

Equivalently,

$$\mathcal{P}_{\mathcal{D}}^{\text{phys}} = \{X \subseteq M \mid \varnothing \neq X \neq M,\ \partial_{\mathcal{D}} X \neq \varnothing\}.$$

An element $X \in \mathcal{P}_{\mathcal{D}}^{\text{phys}}$ will be called a $\mathcal{D}$-physical object.

Commentary. *Thus proper non-empty $\mathcal{D}$-lightons remain topological objects, but they are not $\mathcal{D}$-physical objects in this interaction-bearing sense, since their $\mathcal{D}$-boundary is empty.*

Explanation. *The subsets X of M with the property*

$$\varnothing = X^\circ \neq X \neq \overline{X} \neq M$$

are called spations, because for an element $x \in X$, if

$$\{x\} \notin \mathcal{D},$$

that is, if x is not an etheron, then

$$\{x\}^\circ = \varnothing.$$

The situation

$$\overline{\{x\}} = M$$

appears only if

$$\mathcal{D} = \{M\},$$

in which case all $\mathcal{D}$-physical objects are undons.

Thus, a point of M may be regarded as a spation.

We shall denote by

$$\mathcal{S} = \{X \in \mathcal{P}(M) \mid \varnothing = X^\circ \neq X \neq \overline{X} \neq M\}$$

the set of *spations*;

$$\mathcal{T} = \{X \in \mathcal{P}(M) \mid \varnothing \neq X^\circ \neq X \neq \overline{X} = M\}$$

the set of *tempons*;

$$\mathcal{K} = \{X \in \mathcal{P}(M) \mid \varnothing \neq X^\circ \neq X \neq \overline{X} \neq M\}$$

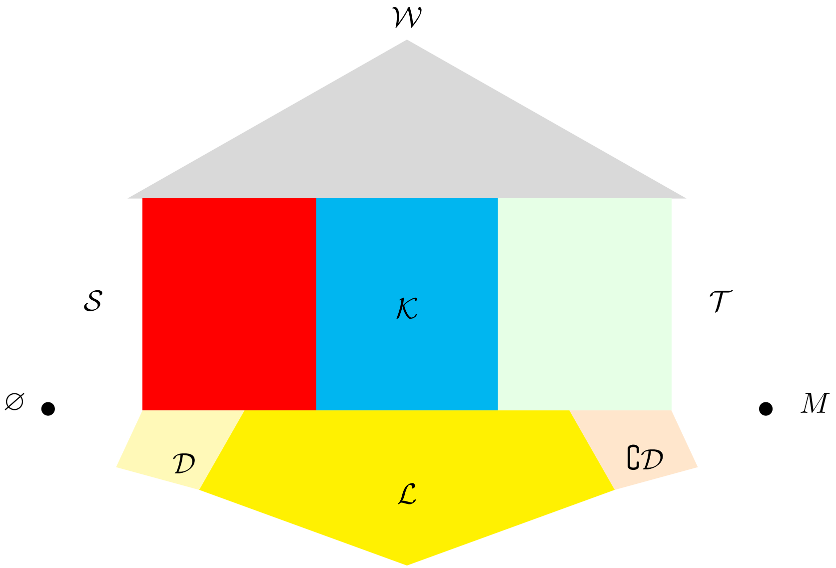
the set of *korpuskons*;

and

$$\mathcal{W} = \{X \in \mathcal{P}(M) \mid \varnothing = X^\circ \neq X \neq \overline{X} = M\}$$

the set of *undons.*

The original manuscript here shows the arrangement of the classes $\mathcal{W}$, $\mathcal{S}$, $\mathcal{T}$, $\mathcal{K}$, lightons, $\mathcal{D}$, and their complements within the interval from $\varnothing$ to M.



1.5 Hidden Symmetry

The “hidden symmetry”

$$\complement_M : \mathcal{P}(M) \to \mathcal{P}(M)$$

is defined by

$$\complement_M X = M \setminus X, \qquad \forall X \in \mathcal{P}(M).$$

It sends $\mathcal{S}$ into $\mathcal{T}$. For this reason, the subsets of M which belong to $\mathcal{T}$ are called *tempons.*

From De Morgan’s laws, we have

$$\varnothing \subseteq X^\circ \subseteq X \subseteq \overline{X} \subseteq M$$

which implies

$$\varnothing \subseteq \complement_M(\overline{X}) \subseteq \complement_M(X) \subseteq \complement_M(X^\circ) \subseteq M.$$

But $\complement_M(\overline{X})$ is the interior of $\complement_M X$, and $\complement_M(X^\circ)$ is the closure of $\complement_M X$. Thus we have the following table.

X	$\complement_M X$
$\mathcal{K}$	$\mathcal{K}$
$\mathcal{T}$	$\mathcal{S}$
$[\mathcal{D}]$	$[\mathcal{D}]$
M	$\varnothing$
$\mathcal{S}$	$\mathcal{T}$
$\mathcal{W}$	$\mathcal{W}$
$\varnothing$	M

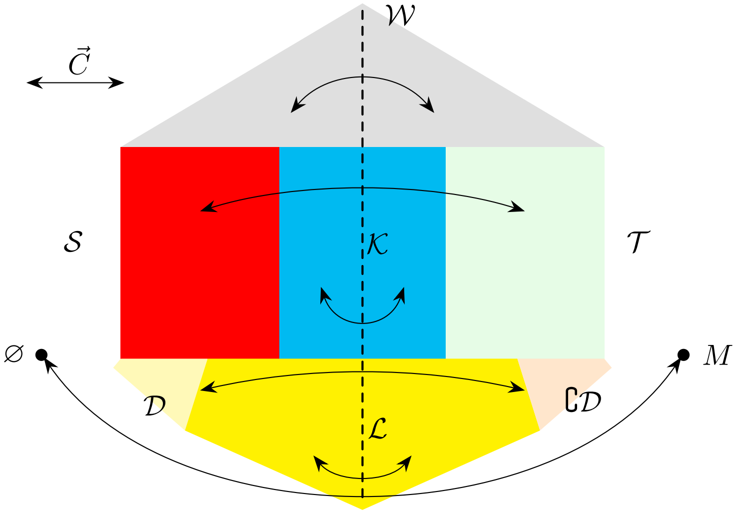
The operator $\complement_M$ is indeed a symmetry, since it is a bijection of $\mathcal{P}(M)$ with the property

$$(\complement_M \circ \complement_M)(X) = \complement_M(\complement_M X) = X, \qquad \forall X \in \mathcal{P}(M).$$

Moreover, it has no fixed points.

The sets of korpuskons, lightons, and undons are invariant under this symmetry, because the symmetric image of a korpuskon is again a korpuskon, although of course a different one, and analogously for the subsets in $[\mathcal{D}]$, respectively for the subsets in $\mathcal{W}$.

The original manuscript here shows the hidden symmetry $\complement_M$, exchanging $\mathcal{S}$ with $\mathcal{T}$, while leaving $\mathcal{K}$, $\mathcal{W}$, and the class of lightons invariant.



1.6 A Morley-Michelson Type Result

Proposition 1.2 (Original numbering: 1.5.1)**.** *If there exists*

$$\{a\} \in \mathcal{D},$$

then

$$\mathcal{W} = \varnothing.$$

Proof. Assume that there exists

$$\{a\} \in \mathcal{D}$$

and that there also exists

$$X \subset M$$

such that

$$\varnothing = X^{\circ} \neq X \neq \overline{X} = M.$$

If

$$\{a\} \subseteq X,$$

then

$$X^{\circ} \neq \varnothing,$$

which is a contradiction.

If

$$\{a\} \cap X = \varnothing,$$

then

$$\overline{X} \neq M,$$

which is again a contradiction.

Thus

$$\mathcal{W} = \varnothing.$$

The proposition is proved. □

Commentary. *In ordinary language, this proposition says that if there exists at least one etheron, then there are no undons. This resembles the result of the Morley-Michelson experiment.*

The converse is also true.

2 "Analysis" of the Universe

2.1 Boundary Operators

The boundary operators are defined in the classical way, as the difference between the closure and the interior of the same set.

Definition 2.1 (Original numbering: 2.1.1)**.** *For every* $X \in \mathcal{P}(M)$*, define*

$$\partial_{\mathcal{D}} X = \overline{\pi}_{\mathcal{D}}(X) \setminus \mathring{\pi}_{\mathcal{D}}(X) = \overline{X} \setminus X^{\circ}.$$

Commentary. *Since* $\overline{X}$ *is also open, and* X° *is also closed,* $\partial_{\mathcal{D}} X$ *is both open and closed. Thus*

$$\partial_{\mathcal{D}} X \in [\mathcal{D}].$$

It is easy to show that

$$\partial_{\mathcal{D}} X = \varnothing \qquad \text{for} \qquad X \in [\mathcal{D}],$$

and

$$\partial_{\mathcal{D}} X = M \qquad \text{for} \qquad X \in \mathcal{W}.$$

Therefore, the subsets of M *for which* $\partial_{\mathcal{D}} X$ *is relevant, that is, for which*

$$\partial_{\mathcal{D}} X \neq \varnothing \qquad \text{and} \qquad \partial_{\mathcal{D}} X \neq M,$$

are only the korpuskons, spations, and tempons:

$$\mathcal{K}, \qquad \mathcal{S}, \qquad \mathcal{T}.$$

Observation. *For a* $\mathcal{D}$*-photon* D *and a subset* $X \subseteq M$*, only the following three situations are possible:*

$$\text{(i)} \qquad D \subseteq X, \quad \text{hence} \quad D \subseteq X^{\circ}.$$

$$\text{(ii)} \qquad D \cap X \neq \varnothing \quad \text{and} \quad D \cap \complement_M X \neq \varnothing, \quad \text{hence} \quad D \subseteq \partial_{\mathcal{D}} X.$$

$$\text{(iii)} \qquad D \subseteq \complement_M X, \quad \text{hence} \quad D \subseteq \mathring{\pi}_{\mathcal{D}}(\complement_M X) = \complement_M \overline{X},$$

and therefore

$$D \cap \overline{X} = \varnothing.$$

Exactly one of these situations occurs. This observation is useful for proofs.

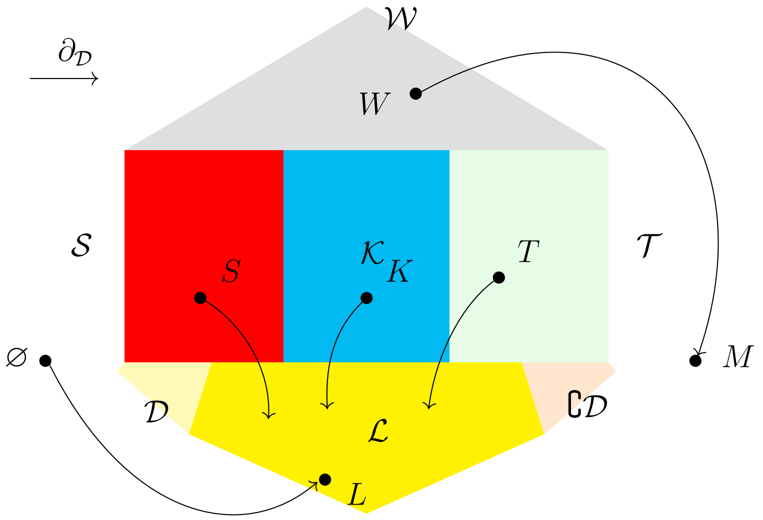
Lemma 2.1 (Original numbering: 2.1.1)**.** *For every* $X \in \mathcal{P}(M)$*,*

$$\partial_{\mathcal{D}} X = \cup[\{D \in \mathcal{D} \mid D \cap X \neq \varnothing \text{ and } D \cap \complement_M X \neq \varnothing\}].$$

Proof. This follows immediately from the preceding observation. □

Commentary. *Lemma 2.1.1 shows that the boundary of a subset* X *is formed from those* $\mathcal{D}$*-photons* D *which intersect both* X *and its complement.*

The original manuscript here illustrates the action of the topological operator $\partial_{\mathcal{D}}$ on $\mathcal{P}(M)$, showing how the classes $\mathcal{S}$, $\mathcal{K}$, $\mathcal{T}$, and $\mathcal{W}$ are mapped under the boundary operation.



Definition 2.2 (Original numbering: 2.1.2)**.** *For every* $X \in \mathcal{P}(M)$*, define the interior boundary and exterior boundary by*

$$\partial^i_{\mathcal{D}} X = X \setminus X^\circ,$$

and

$$\partial^e_{\mathcal{D}} X = \overline{X} \setminus X.$$

Commentary. *The set* $\partial^i_{\mathcal{D}} X$ *will be called the interior boundary of* X*, and the set* $\partial^e_{\mathcal{D}} X$ *will be called the exterior boundary of* X*.*

Evidently,

$$\partial^i_{\mathcal{D}} X \cap \partial^e_{\mathcal{D}} X = \varnothing$$

and

$$\partial^i_{\mathcal{D}} X \cup \partial^e_{\mathcal{D}} X = \partial_{\mathcal{D}} X.$$

While $\partial_{\mathcal{D}} X \in [\mathcal{D}]$*, meaning that it is a union of* $\mathcal{D}$*-photons, one can easily show that* $\partial^i_{\mathcal{D}} K$ *and* $\partial^e_{\mathcal{D}} K$ *are* $\mathcal{D}$*-spations for* $K \in \mathcal{K}$*. More will be said in the following subsection.*

2.2 Interactions

Definition 2.3 (Original numbering: 2.2.1). *Let* $A, B \in \mathcal{P}_{\mathcal{D}}^{\mathrm{phys}}$ *be such that*

$$A \cap B = \varnothing.$$

They $\mathcal{D}$*-interact if and only if*

$$\overline{\pi}_{\mathcal{D}}(A) \cap \overline{\pi}_{\mathcal{D}}(B) \neq \varnothing.$$

Observation. *The condition*

$$A \cap B = \varnothing$$

means that no "material point" of M *belongs to both objects* A *and* B*.*

This is natural, because if

$$A \cap B \neq \varnothing,$$

then the union

$$C = A \cup B$$

will be considered as a single object of study rather than as an interaction pair.

The interaction between two $\mathcal{D}$-physical objects takes place through the field of interaction, as shown in the following proposition.

Proposition 2.1 (Original numbering: 2.2.2). *Let* $A, B \subseteq M$ *be such that* $A \cap B = \varnothing$*. Then*

$$\overline{\pi}_{\mathcal{D}}(A) \cap \overline{\pi}_{\mathcal{D}}(B) = \partial_{\mathcal{D}} A \cap \partial_{\mathcal{D}} B = \cup[\{D \in \mathcal{D} \mid D \cap A \neq \varnothing \text{ and } D \cap B \neq \varnothing\}].$$

Proof. For the first equality, we have

$$\begin{aligned}\overline{\pi}_{\mathcal{D}}(A) \cap \overline{\pi}_{\mathcal{D}}(B) &= \big(A^\circ \cup \partial_{\mathcal{D}} A\big) \cap \big(B^\circ \cup \partial_{\mathcal{D}} B\big) \\ &= \big(A^\circ \cap B^\circ\big) \cup \big(A^\circ \cap \partial_{\mathcal{D}} B\big) \cup \big(\partial_{\mathcal{D}} A \cap B^\circ\big) \cup \big(\partial_{\mathcal{D}} A \cap \partial_{\mathcal{D}} B\big).\end{aligned}$$

We immediately have

$$\varnothing \subseteq A^\circ \cap B^\circ \subseteq A \cap B = \varnothing.$$

Therefore

$$A^\circ \cap B^\circ = \varnothing.$$

Now suppose that

$$A^\circ \cap \partial_{\mathcal{D}} B \neq \varnothing.$$

Since

$$A^\circ \cap \partial_{\mathcal{D}} B \in [\mathcal{D}],$$

there exists $D \in \mathcal{D}$ such that

$$D \subseteq A^\circ \cap \partial_{\mathcal{D}} B.$$

Thus

$$D \subseteq A^\circ$$

and

$$\varnothing \neq D \subseteq \partial_{\mathcal{D}} B.$$

By case (ii) in the observation from Section 2.1, we have

$$D \cap B \neq \varnothing.$$

Choose

$$x \in D \cap B.$$

Then

$$x \in D \subseteq A^\circ \subseteq A,$$

so

$$x \in A \cap B,$$

which contradicts

$$A \cap B = \varnothing.$$

Therefore

$$A^\circ \cap \partial_{\mathcal{D}} B = \varnothing.$$

Analogously,

$$\partial_{\mathcal{D}} A \cap B^\circ = \varnothing.$$

Hence the only remaining term is

$$\partial_{\mathcal{D}} A \cap \partial_{\mathcal{D}} B,$$

and so

$$\overline{\pi}_{\mathcal{D}}(A) \cap \overline{\pi}_{\mathcal{D}}(B) = \partial_{\mathcal{D}} A \cap \partial_{\mathcal{D}} B.$$

The first equality is proved.

It remains to identify the $\mathcal{D}$-photons in this intersection. Let $D \in \mathcal{D}$. Since $A \cap B = \varnothing$, the conditions

$$D \subseteq \partial_{\mathcal{D}} A \cap \partial_{\mathcal{D}} B$$

and

$$D \cap A \neq \varnothing \qquad \text{and} \qquad D \cap B \neq \varnothing$$

are equivalent. Indeed, the first condition implies the second by Lemma 2.1.1. Conversely, if D meets both A and B, then D meets both A and $\complement_M A$, and also both B and $\complement_M B$, because A and B are disjoint. Hence $D \subseteq \partial_{\mathcal{D}} A$ and $D \subseteq \partial_{\mathcal{D}} B$, again by Lemma 2.1.1.

Therefore

$$\partial_{\mathcal{D}} A \cap \partial_{\mathcal{D}} B = \cup[\{D \in \mathcal{D} \mid D \cap A \neq \varnothing \text{ and } D \cap B \neq \varnothing\}],$$

which proves the proposition. □

Observation. *Consequently, if* $A \cap B = \varnothing$ *and*

$$\overline{\pi}_{\mathcal{D}}(A) \cap \overline{\pi}_{\mathcal{D}}(B) \neq \varnothing,$$

then

$$\partial_{\mathcal{D}} A \neq \varnothing \qquad \textit{and} \qquad \partial_{\mathcal{D}} B \neq \varnothing.$$

Thus a proper $\mathcal{D}$*-lighton cannot participate in a non-empty interaction with a disjoint object, because its* $\mathcal{D}$*-boundary is empty.*

Note (Original numbering: 2.2.1)**.** *We write*

$$\mathcal{M}_{\mathcal{D}} = \{X \in \mathcal{P}_{\mathcal{D}}^{\text{phys}} \mid \partial_{\mathcal{D}} X \neq M\} = \mathcal{T} \sqcup \mathcal{S} \sqcup \mathcal{K}.$$

This is the smaller class of $\mathcal{D}$-physical objects whose boundary is neither empty nor total. Undons have maximal boundary, while lightons have zero boundary.

Observation. *For $A, B \in \mathcal{P}_{\mathcal{D}}^{\text{phys}}$, if*

$$\overline{\pi}_{\mathcal{D}}(A) \cap \overline{\pi}_{\mathcal{D}}(B) \neq \varnothing,$$

then the interaction is mediated exactly by those $\mathcal{D}$-photons which meet both objects.

This is in line with Quantum Theory.

Proposition 2.2 (Original numbering: 2.2.3)**.** *For every $X \in \mathcal{M}_{\mathcal{D}}$,*

$$\partial^i_{\mathcal{D}} X \in \mathcal{S} \qquad \text{and} \qquad \partial^e_{\mathcal{D}} X \in \mathcal{S}.$$

Proof. First let $K \in \mathcal{K}$. Thus

$$\varnothing \subsetneq K^\circ \subsetneq K \subsetneq \overline{K} \subsetneq M.$$

We want to show that

$$\partial^i_{\mathcal{D}} K \in \mathcal{S},$$

that is,

$$\varnothing = \mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} K) \subsetneq \partial^i_{\mathcal{D}} K \subsetneq \overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} K) \subsetneq M.$$

Suppose that

$$\mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} K) \neq \varnothing.$$

Then there exists $D \in \mathcal{D}$ such that

$$D \subseteq \mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} K) \subseteq \partial^i_{\mathcal{D}} K = K \setminus K^\circ.$$

Thus

$$D \subseteq K \qquad \text{and} \qquad D \cap K^\circ = \varnothing.$$

Therefore

$$D \cup K^\circ \subseteq K,$$

with

$$K^\circ \subsetneq D \cup K^\circ \in [\mathcal{D}].$$

This contradicts the maximality of K° as the interior of K. Hence

$$\mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} K) = \varnothing.$$

Since

$$\partial^i_{\mathcal{D}} K = K \setminus K^\circ \neq \varnothing,$$

we have

$$\mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} K) \subsetneq \partial^i_{\mathcal{D}} K.$$

Suppose now that

$$\partial^i_{\mathcal{D}} K = \overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} K).$$

But
$$\overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}}K) \in [\mathcal{D}],$$
so by Lemma 1.2.1 we obtain
$$\partial^i_{\mathcal{D}}K = \mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}}K) = \varnothing,$$
which is a contradiction. Therefore
$$\partial^i_{\mathcal{D}}K \subsetneqq \overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}}K).$$

Since
$$\partial^i_{\mathcal{D}}K \subseteq K,$$
by the monotonicity of the closure operator we have
$$\overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}}K) \subseteq \overline{K} \subsetneqq M.$$
Therefore
$$\overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}}K) \subsetneqq M.$$
Thus
$$\partial^i_{\mathcal{D}}K \in \mathcal{S}.$$

Now let $S \in \mathcal{S}$. Thus
$$\varnothing = S^\circ \subsetneqq S \subsetneqq \overline{S} \subsetneqq M.$$
Since
$$\partial^i_{\mathcal{D}}S = S \setminus S^\circ = S,$$
we immediately have
$$\partial^i_{\mathcal{D}}S \in \mathcal{S}.$$

Now let $T \in \mathcal{T}$. Thus
$$\varnothing \subsetneqq T^\circ \subsetneqq T \subsetneqq \overline{T} = M.$$
We want to show that
$$\partial^i_{\mathcal{D}}T \in \mathcal{S},$$
that is,
$$\varnothing = \mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}}T) \subsetneqq \partial^i_{\mathcal{D}}T \subsetneqq \overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}}T) \subsetneqq M.$$

Suppose that
$$\mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}}T) \neq \varnothing.$$
Then there exists $D \in \mathcal{D}$ such that
$$D \subseteq \mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}}T) \subseteq \partial^i_{\mathcal{D}}T = T \setminus T^\circ.$$
Thus
$$D \subseteq T \qquad \text{and} \qquad D \cap T^\circ = \varnothing.$$
Therefore
$$D \cup T^\circ \subseteq T,$$
with
$$T^\circ \subsetneqq D \cup T^\circ \in [\mathcal{D}].$$

This contradicts the maximality of T°. Hence

$$\mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} T) = \varnothing.$$

Since

$$\partial^i_{\mathcal{D}} T = T \setminus T^\circ \neq \varnothing,$$

we have

$$\mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} T) \subsetneq \partial^i_{\mathcal{D}} T.$$

Suppose that

$$\partial^i_{\mathcal{D}} T = \overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} T).$$

But

$$\overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} T) \in [\mathcal{D}],$$

so by Lemma 1.2.1 we obtain

$$\partial^i_{\mathcal{D}} T = \mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} T) = \varnothing,$$

which is a contradiction. Therefore

$$\partial^i_{\mathcal{D}} T \subsetneq \overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} T).$$

Since

$$\partial^i_{\mathcal{D}} T \subseteq \partial_{\mathcal{D}} T \neq \varnothing,$$

by the monotonicity of the closure operator we have

$$\overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} T) \subseteq \overline{\pi}_{\mathcal{D}}(\partial_{\mathcal{D}} T) = \partial_{\mathcal{D}} T \subsetneq M.$$

Therefore

$$\overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} T) \subsetneq M.$$

Thus

$$\partial^i_{\mathcal{D}} T \in \mathcal{S}.$$

Now let $K \in \mathcal{K}$. Thus

$$\varnothing \subsetneq K^\circ \subsetneq K \subsetneq \overline{K} \subsetneq M.$$

We want to show that

$$\partial^e_{\mathcal{D}} K \in \mathcal{S},$$

that is,

$$\varnothing = \mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K) \subsetneq \partial^e_{\mathcal{D}} K \subsetneq \overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K) \subsetneq M.$$

Suppose that

$$\mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K) \neq \varnothing.$$

Then there exists $D \in \mathcal{D}$ such that

$$D \subseteq \mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K) \subseteq \partial^e_{\mathcal{D}} K = \overline{K} \setminus K.$$

Thus

$$D \subseteq \overline{K} \qquad \text{and} \qquad D \cap K = \varnothing.$$

Therefore

$$K \subseteq \overline{K} \setminus D.$$

But

$$\overline{K} \setminus D \in [\mathcal{D}]$$

and

$$\overline{K} \setminus D \subsetneq \overline{K}.$$

This contradicts the minimality of $\overline{K}$ as the closure of K. Hence

$$\mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K) = \varnothing.$$

Since

$$\partial^e_{\mathcal{D}} K = \overline{K} \setminus K \neq \varnothing,$$

we have

$$\mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K) \subsetneq \partial^e_{\mathcal{D}} K.$$

Suppose that

$$\partial^e_{\mathcal{D}} K = \overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K).$$

But

$$\overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K) \in [\mathcal{D}],$$

so by Lemma 1.2.1 we obtain

$$\partial^e_{\mathcal{D}} K = \mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K) = \varnothing,$$

which is a contradiction. Therefore

$$\partial^e_{\mathcal{D}} K \subsetneq \overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K).$$

Since

$$\partial^e_{\mathcal{D}} K \subseteq \partial_{\mathcal{D}} K,$$

by the monotonicity of the closure operator we have

$$\overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K) \subseteq \overline{\pi}_{\mathcal{D}}(\partial_{\mathcal{D}} K) = \partial_{\mathcal{D}} K \subsetneq M.$$

Therefore

$$\overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} K) \subsetneq M.$$

Thus

$$\partial^e_{\mathcal{D}} K \in \mathcal{S}.$$

Now let $T \in \mathcal{T}$. Thus

$$\varnothing \subsetneq T^\circ \subsetneq T \subsetneq \overline{T} = M.$$

We want to show that

$$\partial^e_{\mathcal{D}} T \in \mathcal{S}.$$

But

$$\partial^e_{\mathcal{D}} T = \overline{T} \setminus T = M \setminus T = \complement_M T.$$

By the property of the operator $\complement_M$, established in Section 1.4, we immediately have

$$\partial^e_{\mathcal{D}} T \in \mathcal{S}.$$

Finally, let $S \in \mathcal{S}$. Thus

$$\varnothing = S^\circ \subsetneq S \subsetneq \overline{S} \subsetneq M.$$

We want to show that
$$\partial^e_{\mathcal{D}} S \in \mathcal{S},$$
that is,
$$\varnothing = \mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S) \subsetneq \partial^e_{\mathcal{D}} S \subsetneq \overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S) \subsetneq M.$$

Suppose that
$$\mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S) \neq \varnothing.$$
Then there exists $D \in \mathcal{D}$ such that
$$D \subseteq \mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S) \subseteq \partial^e_{\mathcal{D}} S = \overline{S} \setminus S.$$
Thus
$$D \subseteq \overline{S} \qquad \text{and} \qquad D \cap S = \varnothing.$$
Therefore
$$S \subseteq \overline{S} \setminus D.$$
But
$$\overline{S} \setminus D \in [\mathcal{D}]$$
and
$$\overline{S} \setminus D \subsetneq \overline{S}.$$
This contradicts the minimality of $\overline{S}$. Hence
$$\mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S) = \varnothing.$$

Since
$$\partial^e_{\mathcal{D}} S \neq \varnothing,$$
we have
$$\mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S) \subsetneq \partial^e_{\mathcal{D}} S.$$

Suppose that
$$\partial^e_{\mathcal{D}} S = \overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S).$$
Then, since
$$\overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S) \in [\mathcal{D}],$$
Lemma 1.2.1 gives
$$\partial^e_{\mathcal{D}} S = \mathring{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S) = \varnothing,$$
which is a contradiction. Therefore
$$\partial^e_{\mathcal{D}} S \subsetneq \overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S).$$

Since
$$\partial^e_{\mathcal{D}} S \subseteq \partial_{\mathcal{D}} S,$$
by the monotonicity of the closure operator we have
$$\overline{\pi}_{\mathcal{D}}(\partial^e_{\mathcal{D}} S) \subseteq \overline{\pi}_{\mathcal{D}}(\partial_{\mathcal{D}} S) = \partial_{\mathcal{D}} S \subsetneq M.$$
Therefore all the required inclusions have been proved, and so
$$\partial^e_{\mathcal{D}} S \in \mathcal{S}.$$

Hence, for every $X \in \mathcal{M}_{\mathcal{D}}$,
$$\partial^i_{\mathcal{D}} X \in \mathcal{S} \qquad \text{and} \qquad \partial^e_{\mathcal{D}} X \in \mathcal{S}.$$
The proposition is proved. $\square$

Proposition 2.3 (Original numbering: 2.2.4). *Let A and B be as in Definition 2.2.1. Then*

$$\partial_{\mathcal{D}} A \cap \partial_{\mathcal{D}} B = (\partial^i_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B) \cup (\partial^e_{\mathcal{D}} A \cap \partial^i_{\mathcal{D}} B) \cup (\partial^e_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B),$$

and

$$\partial^i_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B, \qquad \partial^e_{\mathcal{D}} A \cap \partial^i_{\mathcal{D}} B, \qquad \partial^e_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B$$

are spations whenever they are non-empty.

Proof. Since

$$\varnothing \subseteq \partial^i_{\mathcal{D}} A \subseteq A$$

and

$$\varnothing \subseteq \partial^i_{\mathcal{D}} B \subseteq B,$$

while

$$A \cap B = \varnothing,$$

we have

$$\varnothing \subseteq \partial^i_{\mathcal{D}} A \cap \partial^i_{\mathcal{D}} B \subseteq A \cap B = \varnothing.$$

Therefore

$$\partial^i_{\mathcal{D}} A \cap \partial^i_{\mathcal{D}} B = \varnothing.$$

Using

$$\partial_{\mathcal{D}} X = \partial^i_{\mathcal{D}} X \cup \partial^e_{\mathcal{D}} X,$$

we obtain

$$\begin{aligned} \partial_{\mathcal{D}} A \cap \partial_{\mathcal{D}} B &= (\partial^i_{\mathcal{D}} A \cup \partial^e_{\mathcal{D}} A) \cap (\partial^i_{\mathcal{D}} B \cup \partial^e_{\mathcal{D}} B) \\ &= (\partial^i_{\mathcal{D}} A \cap \partial^i_{\mathcal{D}} B) \cup (\partial^i_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B) \cup (\partial^e_{\mathcal{D}} A \cap \partial^i_{\mathcal{D}} B) \cup (\partial^e_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B). \end{aligned}$$

Since the first term is empty, the stated equality follows.

By Proposition 2.2.3 and Lemma 1.2.2, we have

$$\varnothing \subseteq \mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B) \subseteq \mathring{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} A) = \varnothing,$$

and

$$\overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B) \subseteq \overline{\pi}_{\mathcal{D}}(\partial^i_{\mathcal{D}} A) \neq M.$$

Analogous arguments apply to

$$\partial^e_{\mathcal{D}} A \cap \partial^i_{\mathcal{D}} B$$

and

$$\partial^e_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B.$$

Thus the stated components are spations whenever they are non-empty. □

2.3 Classification of Interactions

There are cases in which

$$\partial^e_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B = \varnothing.$$

Such interactions will be called interactions of *gluonic type.*

If

$$\partial^e_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B \neq \varnothing,$$

then the interaction will be called an interaction of *electronic type.*

More precisely:

Definition 2.4 (Original numbering: 2.3.1)**.** *Let A and B be as in Definition 2.2.1, and let $D \in \mathcal{D}$ be such that*

$$D \subseteq \partial_{\mathcal{D}} A \cap \partial_{\mathcal{D}} B.$$

The $\mathcal{D}$-photon D mediates an interaction of electronic type if

$$D \cap (\partial^e_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B) \neq \varnothing.$$

The $\mathcal{D}$-photon D mediates an interaction of gluonic type if

$$D \cap (\partial^e_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B) = \varnothing.$$

Observation. *On the basis of Proposition 2.2.4, the condition*

$$D \cap (\partial^e_{\mathcal{D}} A \cap \partial^e_{\mathcal{D}} B) = \varnothing$$

may be replaced by the equivalent condition

$$D \subseteq \partial^i_{\mathcal{D}} A \cup \partial^i_{\mathcal{D}} B.$$

This condition can be generalised to an arbitrary number of "physical objects".

The meaning of this condition is that the $\mathcal{D}$-photon D is included in

$$A \cup B,$$

although

$$A \cap B = \varnothing,$$

and although

$$D \nsubseteq A, \qquad D \nsubseteq B.$$

It is very interesting that in this case we have

$$\mathring{\pi}_{\mathcal{D}}(A) \cup \mathring{\pi}_{\mathcal{D}}(B) \subsetneq \mathring{\pi}_{\mathcal{D}}(A \cup B).$$

In ordinary language, the interior of the union of the sets A and B is strictly larger, with respect to inclusion, than the union of the interiors of A and B.

This phenomenon will be called synergy and will be analysed later.

Moreover, for a $\mathcal{D}$-photon

$$D \subseteq \partial_{\mathcal{D}} X,$$

we have

$$D_i = D \cap \partial^i_{\mathcal{D}} X, \qquad D_e = D \cap \partial^e_{\mathcal{D}} X.$$

These are spations of a special type, provided that $X \in \mathcal{M}_{\mathcal{D}}$.

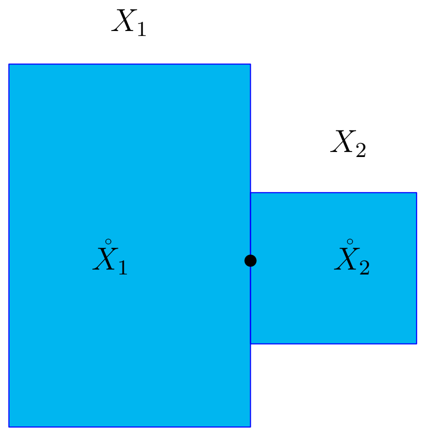
What is stranger is the fact that "synergy" appears even in the Euclidean plane, with the usual topology. The following image shows this phenomenon.

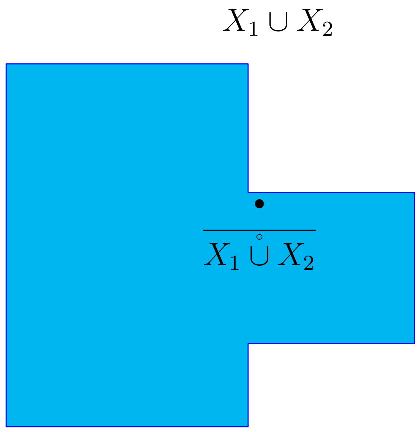
The original manuscript here shows two subsets X_1 and X_2 in the Euclidean plane, together with their interiors X_1° and X_2°, and the interior of their union

$$(X_1 \cup X_2)^\circ.$$

The diagram illustrates that the interior of the union may be strictly larger than the union of the interiors:

$$X_1^\circ \cup X_2^\circ \subsetneq (X_1 \cup X_2)^\circ.$$





This phenomenon resembles what is known in physics as the "mass defect", but here it occurs only at the topological level. A measurable form will be presented in Chapter 3 of the book.

Conceptually, there may be three types of interaction between two "physical objects":

(a) *totally gluonic*, if all $\mathcal{D}$-photons mediate a gluonic interaction;

(b) *totally electronic*, if all $\mathcal{D}$-photons mediate an electronic interaction;

(c) *electro-gluonic*, if some $\mathcal{D}$-photons mediate a gluonic interaction and other $\mathcal{D}$-photons mediate an electronic interaction.

2.4 The Topological Operators $\xi_{\mathcal{D}}$ and $\theta_{\mathcal{D}}$. Quantons and Broglions

We define the topological operator

$$\xi_{\mathcal{D}} : \mathcal{M}_{\mathcal{D}} \to \mathcal{M}_{\mathcal{D}}$$

by

$$\forall X \in \mathcal{M}_{\mathcal{D}}, \qquad \xi_{\mathcal{D}}(X) = \partial^i_{\mathcal{D}} X.$$

We call this the *spatial operator*.

By Proposition 2.2.3, we have

$$\forall X \in \mathcal{M}_{\mathcal{D}}, \qquad \xi_{\mathcal{D}}(X) \in \mathcal{S}.$$

Also,

$$\forall S \in \mathcal{S}, \qquad \xi_{\mathcal{D}}(S) = S.$$

That is, the operator $\xi_{\mathcal{D}}$ leaves the set of spations $\mathcal{S}$ pointwise invariant.

We define the topological operator

$$\theta_{\mathcal{D}} : \mathcal{M}_{\mathcal{D}} \to \mathcal{M}_{\mathcal{D}}$$

by

$$\forall X \in \mathcal{M}_{\mathcal{D}}, \qquad \theta_{\mathcal{D}}(X) = \complement_M(\partial^e_{\mathcal{D}} X).$$

We call this the *temporal operator.*

By Proposition 2.2.3, we have

$$\forall X \in \mathcal{M}_{\mathcal{D}}, \qquad \theta_{\mathcal{D}}(X) \in \mathcal{T}.$$

Also,

$$\forall T \in \mathcal{T}, \qquad \theta_{\mathcal{D}}(T) = T.$$

That is, the operator $\theta_{\mathcal{D}}$ leaves the set of tempons $\mathcal{T}$ pointwise invariant.

The definition of the operators $\xi_{\mathcal{D}}$ and $\theta_{\mathcal{D}}$ can be extended to the set of lightons, because

$$\partial^i_{\mathcal{D}} L = \partial^e_{\mathcal{D}} L = \partial_{\mathcal{D}} L = \varnothing.$$

Thus one could define

$$\xi_{\mathcal{D}}(L) = \varnothing, \qquad \theta_{\mathcal{D}}(L) = M,$$

by considering, as exceptions, that $\varnothing$ is also a spation and that M is also a tempon.

However, this would lead to unnecessary complications.

By contrast, this method cannot be used to extend the definition of $\xi_{\mathcal{D}}$ and $\theta_{\mathcal{D}}$ to undons.

The *quantonic* character, respectively the *broglionic* character, refers to $\partial_{\mathcal{D}} X$, the total field of interaction, and applies only to objects in $\mathcal{M}_{\mathcal{D}}$.

If $\partial_{\mathcal{D}} X$ is minimal, the object X will be called *quantonic.* If $\partial_{\mathcal{D}} X$ is maximal, the object X will be called *broglionic.*

Since, for objects in $\mathcal{M}_{\mathcal{D}}$, we have

$$\varnothing \neq \partial_{\mathcal{D}} X \neq M,$$

the smallest value which $\partial_{\mathcal{D}} X$ can take is

$$\partial_{\mathcal{D}} X \in \mathcal{D},$$

and the largest value is

$$\partial_{\mathcal{D}} X \in \complement_M \mathcal{D},$$

where

$$\complement_M \mathcal{D} := \{\complement_M D \mid D \in \mathcal{D}\}.$$

Thus, for a quantonic spation, we have

$$\varnothing = \mathring{\pi}_{\mathcal{D}}(S), \qquad D = \partial_{\mathcal{D}} S,$$

and therefore

$$D = \overline{\pi}_{\mathcal{D}}(S).$$

For a quantonic tempon, we have

$$\overline{\pi}_{\mathcal{D}}(T) = M, \qquad D = \partial_{\mathcal{D}} T,$$

and therefore

$$\mathring{\pi}_{\mathcal{D}}(T) \in \complement_M \mathcal{D}.$$

By contrast, a broglionic spation has

$$\varnothing = \mathring{\pi}_{\mathcal{D}}(S), \qquad \partial_{\mathcal{D}} S \in \complement_M \mathcal{D},$$

and therefore

$$\overline{\pi}_{\mathcal{D}}(S) \in \complement_M \mathcal{D}.$$

For a broglionic tempon, we have

$$\overline{\pi}_{\mathcal{D}}(T) = M, \qquad \partial_{\mathcal{D}} T \in \complement_M \mathcal{D},$$

and therefore

$$\mathring{\pi}_{\mathcal{D}}(T) \in \mathcal{D}.$$

The situation becomes more complicated in the case of korpuskons. Since

$$\overline{\pi}_{\mathcal{D}}(X) = \mathring{\pi}_{\mathcal{D}}(X) \cup \partial_{\mathcal{D}} X,$$

we may call a korpuskon satisfying

$$\mathring{\pi}_{\mathcal{D}}(K) = D_1 \in \mathcal{D}, \qquad \partial_{\mathcal{D}} K = D_2 \in \mathcal{D}$$

with minimal interior and minimal field of interaction an *elementary korpuskon.* It could also be called a *quantonic korpuskon of spationic type.*

We introduce the notation

$$\widetilde{\mathcal{D}} = \{L \in [\mathcal{D}] \mid L = D_1 \cup D_2, \quad D_1, D_2 \in \mathcal{D}, \quad D_1 \neq D_2\}.$$

The set $\widetilde{\mathcal{D}}$ consists of those lightons which are the union of two distinct $\mathcal{D}$-photons.

It is easy to show that, for an elementary korpuskon,

$$\overline{\pi}_{\mathcal{D}}(K) \in \widetilde{\mathcal{D}}.$$

For a korpuskon satisfying

$$\mathring{\pi}_{\mathcal{D}}(K) \in \complement_M \widetilde{\mathcal{D}}, \qquad \partial_{\mathcal{D}} K = D \in \mathcal{D},$$

that is, with maximal interior and minimal field of interaction, we have

$$\overline{\pi}_{\mathcal{D}}(K) \in \complement_M \mathcal{D}.$$

Such an object could be called a *quantonic korpuskon of temponic type.*

There is also the situation

$$\mathring{\pi}_{\mathcal{D}}(K) = D \in \mathcal{D}, \qquad \overline{\pi}_{\mathcal{D}}(K) \in \complement_M \mathcal{D},$$

that is, with minimal interior and maximal field of interaction, because

$$\partial_{\mathcal{D}} K \in \complement_M \widetilde{\mathcal{D}}.$$

Such a korpuskon could be called a *broglionic korpuskon*, or a *korpuskon of undonic type.*

2.5 The Synergy Theorem: Topological Form

Let

$$\mathcal{X} = \{X_i \mid i \in I\}$$

be a non-empty family of "physical objects" in the universe

$$\mathbb{U} = (M, \mathcal{D}),$$

and let

$$X = \bigcup_{i \in I} X_i.$$

While it is easy to show that

$$\overline{\pi}_{\mathcal{D}}(X) = \bigcup_{i \in I} \overline{\pi}_{\mathcal{D}}(X_i),$$

and that

$$\mathring{\pi}_{\mathcal{D}}(X) \supseteq \bigcup_{i \in I} \mathring{\pi}_{\mathcal{D}}(X_i),$$

equality in the latter inclusion occurs only under special conditions. This is the purpose of the following theorem.

Theorem 2.1 (Original numbering: 2.5.1)**.** *Let*

$$\mathcal{X} = \{X_i \mid i \in I\}$$

be a non-empty family of non-empty subsets of M, pairwise disjoint, and let

$$X = \bigcup_{i \in I} X_i.$$

Let also

$$\mathcal{X}^\circ = \{\mathring{\pi}_{\mathcal{D}}(X_i) \mid i \in I\},$$

which are also pairwise disjoint.

Then

$$X^\circ \setminus \bigcup_{i \in I} \mathring{\pi}_{\mathcal{D}}(X_i) \neq \varnothing$$

if and only if there exist

$$D \in \mathcal{D}$$

and

$$J \subseteq I, \qquad |J| \geq 2,$$

such that

$$D \subseteq \bigcup_{j \in J} \partial^i_{\mathcal{D}} X_j.$$

Proof. Assume first that

$$X^\circ \setminus \bigcup_{i \in I} \mathring{\pi}_{\mathcal{D}}(X_i) \neq \varnothing.$$

Since $\mathcal{D}$ is a partition of M, there exists $D \in \mathcal{D}$ such that

$$D \subseteq X^\circ$$

and

$$D \cap \mathring{\pi}_{\mathcal{D}}(X_i) = \varnothing, \qquad \forall i \in I.$$

Therefore

$$D \subseteq X^\circ \subseteq X.$$

Let

$$J = \{i \in I \mid D \cap X_i \neq \varnothing\}.$$

We first show that

$$|J| \geq 2.$$

Suppose, for contradiction, that

$$|J| = 1.$$

Then there exists a unique $j_0 \in I$ such that

$$D \cap X_{j_0} \neq \varnothing.$$

Since

$$D \cap \mathring{\pi}_{\mathcal{D}}(X_i) = \varnothing, \qquad \forall i \in I,$$

we have in particular

$$D \cap \mathring{\pi}_{\mathcal{D}}(X_{j_0}) = \varnothing.$$

Also,

$$D \cap X_i = \varnothing, \qquad i \neq j_0.$$

Because

$$D \subseteq X = \bigcup_{i \in I} X_i,$$

it follows that

$$D \subseteq X_{j_0}.$$

But since $D \in \mathcal{D}$, this implies

$$D \subseteq \mathring{\pi}_{\mathcal{D}}(X_{j_0}),$$

which contradicts

$$D \cap \mathring{\pi}_{\mathcal{D}}(X_{j_0}) = \varnothing.$$

Thus

$$|J| \geq 2.$$

For each $j \in J$, write

$$D_j = D \cap X_j.$$

Then

$$D = \bigcup_{j \in J} D_j.$$

Moreover,

$$D \cap \mathring{\pi}_{\mathcal{D}}(X_j) = \varnothing, \qquad \forall j \in J,$$

so

$$D_j \cap \mathring{\pi}_{\mathcal{D}}(X_j) = \varnothing.$$

Hence

$$D_j \subseteq X_j \setminus \mathring{\pi}_{\mathcal{D}}(X_j) = \partial^i_{\mathcal{D}} X_j.$$

Therefore

$$D = \bigcup_{j \in J} D_j \subseteq \bigcup_{j \in J} \partial^i_{\mathcal{D}} X_j.$$

Conversely, assume that there exist $D \in \mathcal{D}$ and $J \subseteq I$, with

$$|J| \geq 2,$$

such that

$$D \subseteq \bigcup_{j \in J} \partial^i_{\mathcal{D}} X_j.$$

Since

$$\partial^i_{\mathcal{D}} X_j = X_j \setminus \mathring{\pi}_{\mathcal{D}}(X_j) \subseteq X_j,$$

we have

$$D \subseteq \bigcup_{j \in J} \partial^i_{\mathcal{D}} X_j \subseteq \bigcup_{j \in J} X_j \subseteq X = \bigcup_{i \in I} X_i.$$

Since $D \in \mathcal{D}$ and $D \subseteq X$, it follows that

$$D \subseteq X^\circ.$$

Also,

$$D \cap \mathring{\pi}_{\mathcal{D}}(X_j) = \varnothing, \qquad j \in J.$$

Because

$$D \subseteq \bigcup_{j \in J} X_j$$

and the family

$$\{X_i \mid i \in I\}$$

is pairwise disjoint, we have

$$D \cap X_i = \varnothing, \qquad i \in I \setminus J.$$

Therefore

$$D \cap \mathring{\pi}_{\mathcal{D}}(X_i) = \varnothing, \qquad \forall i \in I.$$

Thus

$$D \cap \bigcup_{i \in I} \mathring{\pi}_{\mathcal{D}}(X_i) = \varnothing.$$

Consequently,

$$\varnothing \neq D \subseteq X^\circ \setminus \bigcup_{i \in I} \mathring{\pi}_{\mathcal{D}}(X_i).$$

The theorem is proved. □

Observation. *The theorem states that the interior of the union is strictly larger, with respect to inclusion, than the union of the interiors if and only if there are interactions of gluonic type between some of the “physical objects”.*

Equivalently, the interior of the union is equal to the union of the interiors if and only if there are no interactions of gluonic type between the “physical objects” in $\mathcal{X}$.

3 Two Compatible Potentials: “Heating” the Universe

The preceding sections are internal to one fixed potential, represented by a single partition $\mathcal{D}$. The present section belongs conceptually to the part of the theory where two compatible potentials are studied at once.

In Section 1, the definition of a Universe uses a partition

$$\mathcal{D} \in \mathfrak{Par}(M),$$

where $\mathfrak{Par}(M)$ is the lattice of all partitions of the set M.

We now introduce an order relation on $\mathfrak{Par}(M)$, so that two partitions $\mathcal{C}, \mathcal{H} \in \mathfrak{Par}(M)$ can be compared by refinement.

3.1 Ordering Partitions

Definition 3.1 (Original numbering: 3.1.1)**.** *Let*

$$\mathcal{C}, \mathcal{H} \in \mathfrak{Par}(M).$$

The partition $\mathcal{C}$ is said to be less fine than the partition $\mathcal{H}$ if and only if, for every $H \in \mathcal{H}$, there exists $C \in \mathcal{C}$ such that

$$H \subseteq C.$$

Observation. *Instead of saying that “$\mathcal{C}$ is less fine than $\mathcal{H}$”, we may say that “$\mathcal{H}$ is finer than $\mathcal{C}$”. We denote this situation by*

$$\mathcal{C} \preceq \mathcal{H}.$$

The expression “$\mathcal{H}$ is a refinement of $\mathcal{C}$” is also used.

This situation is often encountered in the definition of the Riemann integral, in one or several dimensions.

Proposition 3.1 (Original numbering: 3.1.1)**.** *The relation $\preceq$ is a partial order relation on $\mathfrak{Par}(M)$.*

Proof. First, for every

$$\mathcal{D} \in \mathfrak{Par}(M),$$

we have

$$\mathcal{D} \preceq \mathcal{D}.$$

This is evident.

Second, let

$$\mathcal{C}, \mathcal{H} \in \mathfrak{Par}(M)$$

and suppose that

$$\mathcal{C} \preceq \mathcal{H} \qquad \text{and} \qquad \mathcal{H} \preceq \mathcal{C}.$$

We show that

$$\mathcal{C} = \mathcal{H}.$$

Let $H \in \mathcal{H}$ be arbitrary, but fixed for the moment. Since

$$\mathcal{C} \preceq \mathcal{H},$$

there exists $C \in \mathcal{C}$ such that

$$H \subseteq C.$$

For this C, since

$$\mathcal{H} \preceq \mathcal{C},$$

there exists $H_0 \in \mathcal{H}$ such that

$$C \subseteq H_0.$$

Therefore

$$H \subseteq H_0.$$

But $\mathcal{H}$ is a partition of M, so this implies

$$H_0 = H.$$

Consequently,

$$C = H.$$

Since $C \in \mathcal{C}$, we obtain

$$H \in \mathcal{C}.$$

Because H was chosen arbitrarily in $\mathcal{H}$, it follows that

$$\mathcal{H} \subseteq \mathcal{C}.$$

Similarly, one obtains the reverse inclusion

$$\mathcal{C} \subseteq \mathcal{H}.$$

Therefore

$$\mathcal{C} = \mathcal{H}.$$

Third, let

$$\mathcal{C}, \mathcal{H}, \mathcal{D} \in \mathfrak{Par}(M)$$

and suppose that

$$\mathcal{C} \preceq \mathcal{H} \qquad \text{and} \qquad \mathcal{H} \preceq \mathcal{D}.$$

We show that

$$\mathcal{C} \preceq \mathcal{D}.$$

Let $D \in \mathcal{D}$ be arbitrary, but fixed for the moment. Since

$$\mathcal{H} \preceq \mathcal{D},$$

there exists $H \in \mathcal{H}$ such that

$$D \subseteq H.$$

Using again the definition, since

$$\mathcal{C} \preceq \mathcal{H},$$

there exists $C \in \mathcal{C}$ such that

$$H \subseteq C.$$

Therefore

$$D \subseteq C.$$

Since D was chosen arbitrarily in $\mathcal{D}$, we have

$$\mathcal{C} \preceq \mathcal{D}.$$

The proposition is proved. $\square$

Observation (Original numbering: 3.2)**.** *The ordered set*

$$(\mathfrak{Par}(M), \preceq)$$

is a lattice, with first element

$$\mathrm{Who}(M) = \{M\},$$

the partition with a single component, the "whole M", and with last element

$$\mathrm{Ato}(M) = \{\{x\} \mid x \in M\},$$

the atomic partition, in which each point of M is a component of the partition.

This partition is "totally etheronic".

Observation. *In ordinary language, Definition 3.1.1 requires each $\mathcal{C}$-photon C to be exactly the union of some $\mathcal{H}$-photons.*

If $\mathcal{H}$ is not etheronic, then neither will $\mathcal{C}$ be etheronic. However, it may happen that $\mathcal{C}$ is not etheronic while $\mathcal{H}$ is etheronic.

Also, the operator

$$[\,] : \mathfrak{Par}(M) \to \mathfrak{Top}(M),$$

which associates to each partition

$$\mathcal{D} \in \mathfrak{Par}(M)$$

the closed-open topology

$$[\mathcal{D}] \in \mathfrak{Top}(M),$$

as in Definition 1.2.1, is increasing in the sense that

$$\mathcal{C} \preceq \mathcal{H} \Longrightarrow [\mathcal{C}] \subseteq [\mathcal{H}].$$

For example,

$$[\mathrm{Who}(M)] = \{\varnothing, M\}$$

for the least fine partition of M, and

$$[\mathrm{Ato}(M)] = \mathcal{P}(M) = 2^M$$

for the finest partition of M.

Definition 3.2 (Original numbering: 3.1.2)**.** *The universe*

$$\mathbb{C} = (M, \mathcal{C})$$

is colder than the universe

$$\mathbb{H} = (M, \mathcal{H}),$$

with the same support set M, if and only if

$$\mathcal{C} \preceq \mathcal{H}.$$

3.2 Heating and the Behaviour of Interior, Closure, and Boundary

Proposition 3.2 (Original numbering: 3.2.1)**.** *Let $\mathcal{C}, \mathcal{H}$ be as in Definition 3.1.2. Then, for every $X \in \mathcal{P}(M)$,*

$$\mathring{\pi}_{\mathcal{C}}(X) \subseteq \mathring{\pi}_{\mathcal{H}}(X),$$

and

$$\overline{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X).$$

Proof. First, if

$$\mathring{\pi}_{\mathcal{C}}(X) = \varnothing,$$

there is nothing to prove. So assume that

$$\mathring{\pi}_{\mathcal{C}}(X) \neq \varnothing.$$

Let

$$x \in \mathring{\pi}_{\mathcal{C}}(X)$$

be arbitrary, but fixed for the moment.

Since

$$\mathring{\pi}_{\mathcal{C}}(X) \in [\mathcal{C}],$$

there exists $C \in \mathcal{C}$ such that

$$x \in C \subseteq \mathring{\pi}_{\mathcal{C}}(X) \subseteq X.$$

Since $\mathcal{H}$ is a partition of M, there exists $H \in \mathcal{H}$ such that

$$x \in H.$$

Thus

$$x \in H \cap C.$$

Because

$$\mathcal{C} \preceq \mathcal{H},$$

we have

$$H \subseteq C.$$

Therefore

$$x \in H \subseteq C \subseteq \mathring{\pi}_{\mathcal{C}}(X) \subseteq X.$$

Hence

$$x \in \mathring{\pi}_{\mathcal{H}}(X).$$

Since x was chosen arbitrarily in $\mathring{\pi}_{\mathcal{C}}(X)$, it follows that

$$\mathring{\pi}_{\mathcal{C}}(X) \subseteq \mathring{\pi}_{\mathcal{H}}(X).$$

Now let

$$x \in \overline{\pi}_{\mathcal{H}}(X)$$

be arbitrary, but fixed for the moment.

Since

$$\overline{\pi}_{\mathcal{H}}(X) \in [\mathcal{H}],$$

and since $\mathcal{H}$ is a partition of M, there exists $H \in \mathcal{H}$ such that

$$x \in H \subseteq \overline{\pi}_{\mathcal{H}}(X).$$

Therefore

$$H \cap X \neq \varnothing.$$

Since $\mathcal{C}$ is a partition of M, there exists $C \in \mathcal{C}$ such that

$$x \in C.$$

Thus

$$x \in C \cap H.$$

Because

$$\mathcal{C} \preceq \mathcal{H},$$

we have

$$H \subseteq C.$$

Therefore

$$\varnothing \neq H \cap X \subseteq C \cap X.$$

So

$$C \cap X \neq \varnothing.$$

Hence

$$C \subseteq \overline{\pi}_{\mathcal{C}}(X).$$

Since

$$x \in C,$$

we obtain

$$x \in \overline{\pi}_{\mathcal{C}}(X).$$

Since x was chosen arbitrarily in $\overline{\pi}_{\mathcal{H}}(X)$, it follows that

$$\overline{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X).$$

The proposition is proved. □

Corollary 3.1 (Original numbering: 3.2.1)**.** *Let $\mathcal{C}, \mathcal{H}$ be as in Definition 3.1.2. Then, for every $X \in \mathcal{P}(M)$,*

$$\partial_{\mathcal{H}} X \subseteq \partial_{\mathcal{C}} X.$$

Proof. Using the preceding proposition and De Morgan's laws, we have

$$\partial_{\mathcal{H}} X = \overline{\pi}_{\mathcal{H}}(X) \setminus \mathring{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X) \setminus \mathring{\pi}_{\mathcal{C}}(X) = \partial_{\mathcal{C}} X.$$

□

Corollary 3.2 (Original numbering: 3.2.2)**.** *Let $\mathcal{C}, \mathcal{H}$ be as in Definition 3.1.2. Then, for every $X \in \mathcal{P}(M)$,*

$$\partial^i_{\mathcal{H}} X \subseteq \partial^i_{\mathcal{C}} X$$

and

$$\partial^e_{\mathcal{H}} X \subseteq \partial^e_{\mathcal{C}} X.$$

Proof. First,

$$\partial^i_{\mathcal{H}} X = X \cap \partial_{\mathcal{H}} X \subseteq X \cap \partial_{\mathcal{C}} X = \partial^i_{\mathcal{C}} X.$$

Then, using Proposition 3.2.1,

$$\partial^e_{\mathcal{H}} X = \overline{\pi}_{\mathcal{H}}(X) \setminus X \subseteq \overline{\pi}_{\mathcal{C}}(X) \setminus X = \partial^e_{\mathcal{C}} X.$$

□

Observation. *In ordinary language, Proposition 3.2.1 says that the interior of X "grows", while the closure of X "decreases", with respect to inclusion.*

Corollary 3.2.1 says that the boundary of X also decreases, with respect to inclusion.

It now becomes clear why this phenomenon has been called "heating": because the "field of interaction"

$$\partial_{\mathcal{D}}$$

decreases. Thus, two "physical objects" A and B which interacted in the colder universe $\mathbb{C}$ may no longer interact in the warmer universe $\mathbb{H}$.

Moreover, an interaction of gluonic type may become one of electronic type, or the number of gluonic-type photons mediating the interaction may decrease. This is similar to the transformation of the strong nuclear force into the weak nuclear force.

Another example could be interpreted as the reduction of the force of cohesion between molecules, as in the transformation of water from ice into liquid, steam, and eventually plasma.

Corollary 3.2.2 may have an interesting interpretation: the "proper spation" decreases with respect to inclusion, that is,

$$\xi_{\mathcal{H}}(X) \subseteq \xi_{\mathcal{C}}(X),$$

while the "proper tempon" increases with respect to inclusion, that is,

$$\theta_{\mathcal{H}}(X) \supseteq \theta_{\mathcal{C}}(X).$$

This is similar to the well-known phenomenon in relativity theory.

Moreover, the same phenomenon also occurs for the "external spation"

$$\partial^e_{\mathcal{D}} X$$

and the "external tempon"

$$\complement_M(\partial^e_{\mathcal{D}} X).$$

Now, for every $X \in \mathcal{P}(M)$, we have

$$\varnothing \subseteq \mathring{\pi}_{\mathcal{C}}(X) \subseteq \mathring{\pi}_{\mathcal{H}}(X) \subseteq X \subseteq \overline{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X) \subseteq M.$$

Let us analyse what happens to a subset X when we change from $\mathcal{C}$ to $\mathcal{H}$.

For $\varnothing$ and M, it is clear that their status does not change.

For $D \in \mathcal{C}$, it is clear that

$$D \in [\mathcal{H}],$$

because a $\mathcal{C}$-photon is decomposed into one or more $\mathcal{H}$-photons, and therefore becomes an $\mathcal{H}$-lighton.

Moreover, if

$$D \in [\mathcal{C}],$$

then

$$D \in [\mathcal{H}].$$

This means that a $\mathcal{C}$-lighton becomes an $\mathcal{H}$-lighton.

Therefore, what remains to be analysed are the classes

$$\mathcal{S}, \qquad \mathcal{T}, \qquad \mathcal{K}, \qquad \mathcal{W}.$$

Since $[\mathcal{H}]$ is also a closed-open topology, the cases

$$\mathring{\pi}_{\mathcal{H}}(X) = X \neq \overline{\pi}_{\mathcal{H}}(X)$$

and

$$\mathring{\pi}_{\mathcal{H}}(X) \neq X = \overline{\pi}_{\mathcal{H}}(X)$$

are impossible.

Thus only the following two situations remain possible:

$$\varnothing \subseteq \mathring{\pi}_{\mathcal{C}}(X) \subseteq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X) \subseteq M, \tag{1}$$

and

$$\varnothing \subseteq \mathring{\pi}_{\mathcal{C}}(X) \subseteq \mathring{\pi}_{\mathcal{H}}(X) = X = \overline{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X) \subseteq M. \tag{2}$$

Proposition 3.3 (Original numbering: 3.2.2)**.** *Let $\mathcal{C}, \mathcal{H}$ be as in Definition 3.1.2. Then:*

1. *A $\mathcal{C}$-korpuskon may become an $\mathcal{H}$-korpuskon or an $\mathcal{H}$-lighton.*
2. *A $\mathcal{C}$-tempon may become an $\mathcal{H}$-korpuskon, an $\mathcal{H}$-tempon, or an $\mathcal{H}$-lighton.*
3. *A $\mathcal{C}$-spation may become an $\mathcal{H}$-korpuskon, an $\mathcal{H}$-spation, or an $\mathcal{H}$-lighton.*
4. *A $\mathcal{C}$-undon may become an $\mathcal{H}$-korpuskon, an $\mathcal{H}$-spation, an $\mathcal{H}$-tempon, an $\mathcal{H}$-undon, or an $\mathcal{H}$-lighton.*

Proof. Relation (2) shows that if X is a $\mathcal{C}$-korpuskon, $\mathcal{C}$-tempon, $\mathcal{C}$-spation, or $\mathcal{C}$-undon, then X may become an $\mathcal{H}$-lighton.

Now consider relation (1).

1. Let X be a $\mathcal{C}$-korpuskon. Then

$$\varnothing \neq \mathring{\pi}_{\mathcal{C}}(X) \subseteq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X) \neq M.$$

Hence

$$\varnothing \neq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \neq M,$$

so X is an $\mathcal{H}$-korpuskon.

2. Let X be a $\mathcal{C}$-tempon. Then

$$\varnothing \neq \mathring{\pi}_{\mathcal{C}}(X) \subseteq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X) = M.$$

If

$$\overline{\pi}_{\mathcal{H}}(X) = \overline{\pi}_{\mathcal{C}}(X),$$

then

$$\varnothing \neq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) = M,$$

so X is an $\mathcal{H}$-tempon.

If instead

$$\overline{\pi}_{\mathcal{H}}(X) \neq \overline{\pi}_{\mathcal{C}}(X),$$

then

$$\varnothing \neq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \neq M,$$

so X becomes an $\mathcal{H}$-korpuskon.

3. Let X be a $\mathcal{C}$-spation. Then

$$\varnothing = \mathring{\pi}_{\mathcal{C}}(X) \subseteq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X) \neq M.$$

If

$$\mathring{\pi}_{\mathcal{C}}(X) = \mathring{\pi}_{\mathcal{H}}(X),$$

then

$$\varnothing = \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \neq M,$$

so X becomes an $\mathcal{H}$-spation.

If instead

$$\mathring{\pi}_{\mathcal{C}}(X) \neq \mathring{\pi}_{\mathcal{H}}(X),$$

then

$$\varnothing \neq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \neq M,$$

so X becomes an $\mathcal{H}$-korpuskon.

4. Let X be a $\mathcal{C}$-undon. Then

$$\varnothing = \mathring{\pi}_{\mathcal{C}}(X) \subseteq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X) = M.$$

If

$$\mathring{\pi}_{\mathcal{C}}(X) = \mathring{\pi}_{\mathcal{H}}(X),$$

then

$$\varnothing = \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \subseteq M.$$

If

$$\overline{\pi}_{\mathcal{H}}(X) = \overline{\pi}_{\mathcal{C}}(X),$$

then

$$\varnothing = \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) = M,$$

so X is an $\mathcal{H}$-undon.

If

$$\overline{\pi}_{\mathcal{H}}(X) \neq \overline{\pi}_{\mathcal{C}}(X),$$

then

$$\varnothing = \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \neq M,$$

so X is an $\mathcal{H}$-spation.

If

$$\mathring{\pi}_{\mathcal{C}}(X) \neq \mathring{\pi}_{\mathcal{H}}(X),$$

then

$$\varnothing \neq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \subseteq \overline{\pi}_{\mathcal{C}}(X) = M.$$

If

$$\overline{\pi}_{\mathcal{H}}(X) = \overline{\pi}_{\mathcal{C}}(X),$$

then

$$\varnothing \neq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) = M,$$

so X is an $\mathcal{H}$-tempon.

If

$$\overline{\pi}_{\mathcal{H}}(X) \neq \overline{\pi}_{\mathcal{C}}(X),$$

then

$$\varnothing \neq \mathring{\pi}_{\mathcal{H}}(X) \neq X \neq \overline{\pi}_{\mathcal{H}}(X) \neq M,$$

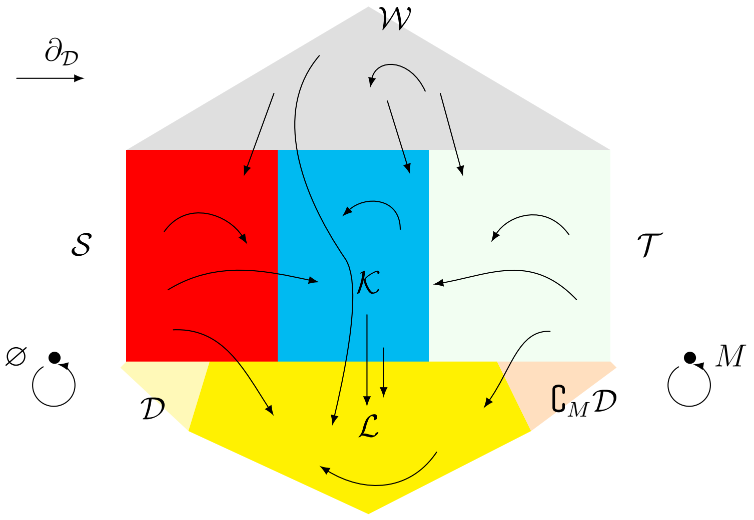
so X is an $\mathcal{H}$-korpuskon.

The proposition is proved. □

The following diagram shows the transformation of the different types of physical objects in passing from the colder universe $\mathbb{C}$ to the warmer universe $\mathbb{H}$.

The original manuscript here shows how the classes $\mathcal{W}$, $\mathcal{S}$, $\mathcal{T}$, $\mathcal{K}$, lightons, $\mathcal{D}$, and their complements transform when passing from the colder universe $\mathbb{C}$ to the warmer universe $\mathbb{H}$.

Placeholder for the manuscript diagram.



4 A Possible Physical Interpretation

The mathematical theory presented in this material is offered without the guarantee that it will solve particular problems in physics, but with the hope that it may be useful.

Of course, the physical interpretation may differ from reader to reader. The author confesses that he does not expect unanimous approval.

There are several open problems.

(1) Although it is easy to prove that, for two objects A and B in $\mathcal{M}_{\mathcal{D}}$, as in Note 2.2.1, we have
$$\xi_{\mathcal{D}}(A) \neq \xi_{\mathcal{D}}(B),$$
which means that the two objects have different "proper spations", that is, they do not occupy the same space, and also
$$\theta_{\mathcal{D}}(A) \neq \theta_{\mathcal{D}}(B),$$
which means that their "proper tempons" are different, it remains to be analysed whether we have the situation
$$\forall T \in \mathcal{T}, \quad \forall S \in \mathcal{S}, \quad \exists K \in \mathcal{K} \quad \text{such that} \quad \xi_{\mathcal{D}}(K) = S \quad \text{and} \quad \theta_{\mathcal{D}}(K) = T.$$
This problem would be called the "empty space-time problem".

(2) How could one associate, for example, to a "physical object" $X \in \mathcal{P}^{+}(M)$, in a natural and meaningful way, an undon
$$W \in \mathcal{W}.$$
Since we have adopted the principle
$$X \cap W = \varnothing,$$
and since we certainly have
$$\overline{X} \cap \overline{W} \neq \varnothing,$$
because
$$\overline{W} = M,$$
one idea would be
$$\overline{X} \cap W \neq \varnothing.$$
This is possible only if
$$X^{\circ} = \varnothing,$$
because for every $D \in \mathcal{D}$,
$$D \cap W \neq \varnothing.$$

The author's intention was to regard the korpuskons as bodies, the spations as elements of space $\mathcal{S}$, the tempons as elements of time $\mathcal{T}$, the undons as waves, the $\mathcal{D}$-photons as photons, and the lightons as light beams.

The "quantonic" character for spations, tempons, and korpuskons is determined by the minimality of the "field", or "capacity", of interaction, and the "broglionic" character by its maximality. See Louis de Broglie.

It has already been mentioned that

$$\partial_{\mathcal{D}} X,$$

the total field, or total capacity, of interaction, is formed from $\mathcal{D}$-photons.

It is interesting that if

$$D \subseteq \partial_{\mathcal{D}} X$$

is a $\mathcal{D}$-photon, then

$$D_i = D \cap \partial_{\mathcal{D}}^i X$$

and

$$D_e = D \cap \partial_{\mathcal{D}}^e X$$

are quantonic spations.

Thus, at least for $X \in \mathcal{M}_{\mathcal{D}}$, the "internal field" and the "external field" of interaction are unions of quantonic spations, which is in the spirit of Quantum Field Theory.

Moreover, if

$$D \subseteq \partial_{\mathcal{D}} X$$

is a $\mathcal{D}$-photon and D_e does not intersect any other physical object, then one may consider that D is emitted, or absorbed, at the reader's discretion, by the "physical object" X.

Note. *A model which may help in understanding the theory is a puzzle. The pieces may be regarded as the $\mathcal{D}$-photons, and the images on these pieces as the "physical objects".*

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`www.texmacs.org`

which made writing this material relatively easy.

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Announcement

The present material is "open source", that is, it may be used by anyone.

Of course, I would be glad to receive an e-mail with citations. Moreover, the LaTeX version will be sent to those who wish to translate this work into their native language.

Translation Note

This English translation was prepared by Peter M. Austin, Information Physics Institute, peter.austin@informationphysicsinstitute.net, with assistance from ChatGPT. Key passages were also checked against Google Translate during the translation process. The mathematical notation has been lightly standardised for readability, while the author's technical neologisms have been preserved. The author's subsequent notation corrections have been incorporated in this 2nd version.